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Optimal Number of Neighbors for Fast Consensus in Wireless Networks with Access Delay

Hyun-Ho Choi and Sanghoon Kim
Department of Electrical, Electronic and Control Engineering,
Institute for Information Technology Convergence,
Hankyong National University Anseong 456-749, Korea
E-mail: {hhchoi,kimsh}@hknu.ac.kr

Abstract

A consensus algorithm causes the state of all nodes in a network reach an agreement by allowing interaction only among adjacent nodes in a distributed manner. As the number of cooperating neighboring nodes increases, the consensus algorithm converges faster, but the time required for information sharing in the wireless network with a medium access control (MAC) delay increases due to access collisions. Therefore, in such kind of wireless network, the consensus algorithm needs to establish a tradeoff between the convergence speed and the sharing delay based on the number of cooperating neighbors. In this paper, we obtain the optimal number of neighbors, to minimize the consensus time, according to the network size. We also present the best strategy to operate the consensus algorithm for fast consensus in a practical wireless network with MAC delay.

Key Words: Consensus, Medium access control, Access delay, Optimal number of neighbors, Distributed wireless networks

1. Introduction

In a network of agents, *consensus* means to reach an agreement regarding a certain quantity of interest that depends on the state of all agents. A consensus algorithm, or protocol, is an interaction rule governing information exchange between an agent and all of its neighbors in the network [1]. Consensus algorithms have been applied to various communication and networking areas that require consensus, such as distributed time or frequency synchronization, distributed fair resource allocation, fast distributed data sharing, and distributed fusion of sensing information [2]-[5].

A key performance metric of a consensus algorithm is the speed at which consensus can be achieved [6]-[7]. This is affected by the control parameters of the consensus algorithm and the network environment variables. To this day, studies on consensus algorithms have mainly focused on the existence and speed of convergence, but have not considered the communication time delay that occurs when state information is shared among nodes in a practical network environment with medium access control (MAC) delay [1], [6], [7].

The convergence speed of the consensus algorithms is increasing with the increase in the amount of information shared among nodes (i.e., as the number of cooperating neighboring nodes increases) [4]. Therefore, the ideal solution to minimize the time required to achieve consensus (i.e., consensus time) is to make all the nodes in the network share their state information with all the other nodes (i.e., form an all-to-all network connectivity). However, in a practical wireless network with MAC delay, a communication time delay occurs when a node transmits data to the other nodes. Thus, when a consensus algorithm is employed in a practical network situation, there is delay due to the sharing of information among nodes. As the number of cooperating neighbor nodes increases, this delay increases significantly due to access collisions [8], [9]. Therefore, the overall time required for achieving consensus depends on the number of cooperating neighbors; an optimal number of neighboring nodes can be obtained by establishing a tradeoff between the convergence time and the information sharing delay.

In this paper, we investigate the tradeoff performance of a consensus algorithm based on the number of neighboring nodes in a distributed wireless network that uses slotted ALOHA as the MAC protocol. Thereafter, we derive the optimal number of neighboring nodes to minimize the consensus time and present a method to operate the consensus algorithm according to the network size.

The rest of this paper is organized as follows. We explain the considered consensus algorithm in Section 2. In Section 3, we analyze the consensus time considering the MAC delay in a distributed wireless network. In Section 4, we show the performance of consensus time and obtain the optimal number of neighboring nodes. Finally, Section 5 provides some concluding remarks.

2. Consensus Algorithm

A consensus algorithm can be expressed in the form of several formulas based on the type of operation, i.e., continuous or discrete time, and the processing rule of the received state information [1]. Here, we consider a discrete-time version of the consensus algorithm expressed as

$$x_i(k+1) = \frac{1}{1+N_i}\left(x_i(k) + \sum_{j \in \mathbf{N}_i} x_j(k) \right) \tag{1}$$

where $x_i(k)$ is the state value of node i at the k-th time slot, $\mathbf{N}_i$ denotes the set of neighboring nodes of node i that can receive data from it, and N_i indicates the number of neighbors of node i. Under this consensus algorithm, each node determines the next state value as the average of its own state value and those of its neighbors, in a distributed manner. This iteration gradually makes the state values of all nodes be similar, as k increases and eventually consensus is achieved (i.e., $x_i \fallingdotseq x_j \ \forall\ i, j$). The number of iterations required for convergence depends on the initial state values and the network topology [1]. Therefore, the exact convergence time can be determined by a simulation based on the consensus algorithm described in (1).

3. Consensus Time in Wireless Networks with Access Delay

Because the consensus algorithm requires each node to transmit its state value to its neighboring nodes, transmission delay occurs in an actual wireless network. In particular, this data transmission is repeated in a distributed manner resulting in frequent access collisions; this leads to a significant increase in the MAC delay of the transmitted data. In this paper, we consider the slotted ALOHA protocol as the MAC protocol because of its simplicity and efficiency in transmitting the state information with small and fixed size [10]. We will use this MAC protocol when analyzing the MAC delay due to information sharing and deriving the consensus time.

The considered MAC protocol offers m time slots per frame and the length of the time slot is set to the time duration required to transmit state information of one node. All nodes are synchronized to the start of the MAC frame. Each of them randomly selects one slot among m time slots and transmits its state information in each iteration of the consensus algorithm. If two or more nodes select the same slot and transmit data simultaneously, access collision occurs and the transmission is repeated in the next frame.

When each node has N_i neighbors and all nodes contend to transmit data in every frame, the probability that x arbitrary nodes successfully transmit data is calculated as

$$p(x) = \begin{cases} \dfrac{N_i!\binom{m}{x}\pi(x)}{m^{N_i}} & \text{if } 0 \le x \le \min\{N_i - 2, m-1\}, \\ \dfrac{N_i!\binom{m}{x}}{m^{N_i}} & \text{if } x = N_i \text{ and } m \ge N_i, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The denominator of (2) denotes the total number of cases. The numerator is expressed as the product of $N_i!\binom{m}{x}$, which represents the case where x nodes succeed in accessing some slots from the m time slots, and $\pi(x)$, which denotes the failed attempts of (N_i-x) nodes to access i slots out of the $(m\text{-}x)$ slots that remains after x nodes have succeeded in accessing. It is expressed as

$$\pi(x)=\sum_{i=1}^{\min\left\{\left\lfloor \frac{N_i-x}{2}\right\rfloor, m-x\right\}}\left\{\binom{m-x}{i}\sum_{a_1,a_2,\cdots,a_i\geq 2}^{a_1+a_2+\cdots+a_i=N_i-x}\frac{1}{\prod_{j=1}^{i}a_j!}\right\} \tag{3}$$

Using (2), the average probability of a successful transmission is given by

$$P_s=\frac{1}{N_i}\sum_{x=0}^{N_i}xp(x). \tag{4}$$

The average transmission delay is defined as the time required for successfully transmitting a packet. Therefore, when N_i nodes are contending, the average time required for a node to successfully transmit its data is obtained as

$$D=mT_s\sum_{x=1}^{\infty}x(1-P_s)^{x-1}P_s=\frac{mT_s}{P_s} \tag{5}$$

where T_s is the length of a single time slot.

For the consensus algorithm, all the nodes in a network need to transmit their state information while contending with neighboring nodes. When adjacent N_i neighboring nodes perform this action simultaneously, the average time required to share the information of all nodes is $N_i D$. Consequently, the consensus time, $T_{consensus}$, is expressed as

$$T_{consensus}=N_{iter}N_iD \tag{6}$$

where N_{iter} denotes the number of iterations required for the convergence of the consensus algorithm.

4. Results and Discussion

For the simulation of the considered consensus algorithm, we assumed a regular network topology [1]. We obtained the number of iterations required for convergence of the consensus algorithm based on the number of cooperating neighbors; finally, we obtained the consensus time. The initial state value of each node was randomly selected to be between 0 and 100. The condition of convergence in the consensus algorithm was set to the case that the 2-norm of the difference between current state values and the theoretical convergence value is less than 10^{-2}.

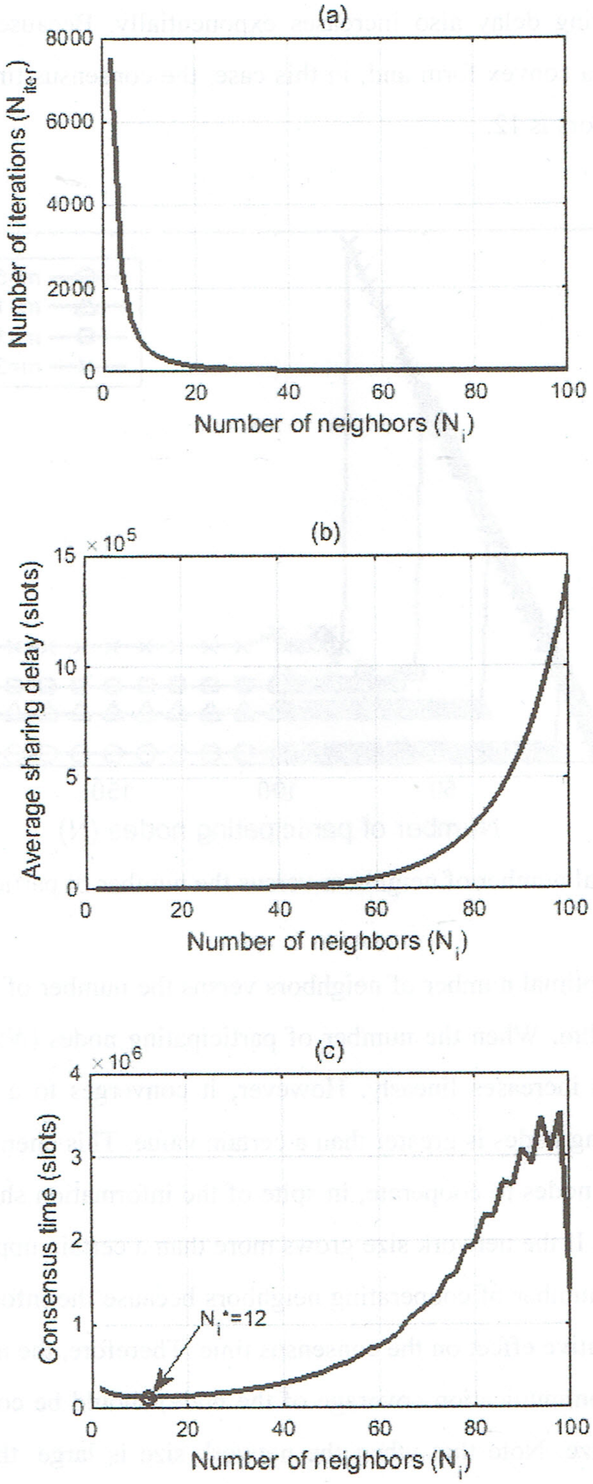

Fig. 1. (a) number of iterations for convergence, (b) average information sharing delay, and (c) consensus time versus the number of neighbors when N=100 and m=15.

Fig. 1 shows the number of iterations (N_{iter}), average information sharing delay (D), and the consensus time ($T_{consensus}$) versus the number of neighbors (N_i) when the number of participating nodes (N) is 100 and the number of time slots per frame (m) is 15. As the number of neighbors increases, the number of iterations decreases exponentially. However, the information sharing delay also increases exponentially. Because of this tradeoff, the consensus time is of a convex form and, in this case, the consensus time is minimized when the number of neighbors is 12.

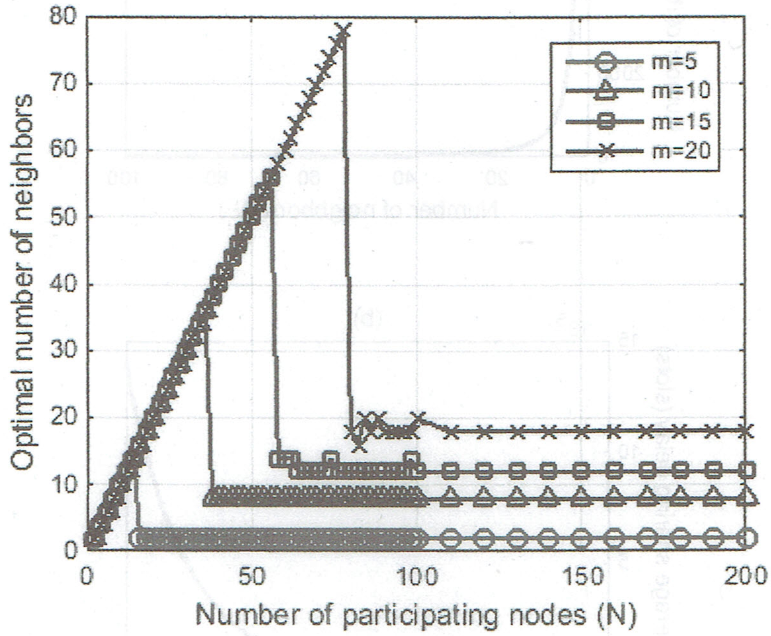

Fig. 2. Optimal number of neighbors versus the number of participating nodes.

Fig. 2 shows the optimal number of neighbors versus the number of nodes participating in the consensus algorithm. When the number of participating nodes (N) is small, the optimal number of neighbors increases linearly. However, it converges to a constant value if the number of participating nodes is greater than a certain value. This phenomenon indicates that it is better for all the nodes to cooperate, in spite of the information sharing delay, when the network size is small. If the network size grows more than a certain upper bound, however, it is better to limit the number of cooperating neighbors because the information sharing delay dominantly has a negative effect on the consensus time. Therefore, the number of cooperating neighbors (i.e., the communication coverage of the node) should be controlled based on the consensus network size. Note that when the network size is large, the optimal number of neighbors converges to a constant value regardless of the number of participating nodes (N) and depends only on the number of time slots (m).

Fig. 3 shows the maximum number of participating nodes for full coverage and the optimal number of neighbors versus the number of time slots allocated. The maximum number of participating nodes for full coverage means that up to this number of participating nodes, the full coverage maintaining all-to-all network connectivity is optimal for fast consensus. As the number of time slots increases, the probability of access collision decreases and information sharing delay decreases. Accordingly, both the optimal number of neighbors and the maximum number of participating nodes for full coverage increases as the number of time slots increases.

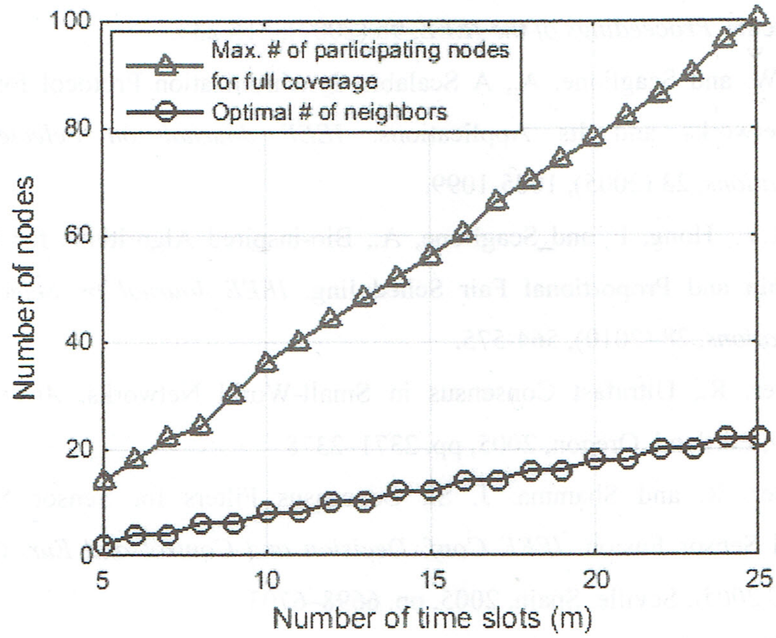

Fig. 3. Maximum number of participating nodes for full coverage and the optimal number of neighbors versus the number of time slots allocated.

5. Conclusions

In this paper, we investigated the performance of a consensus algorithm that establishes a tradeoff between the convergence time and the communication delay based on the number of cooperating neighbors in a practical wireless network with MAC delay. Thereafter, we obtained the optimal number of neighboring nodes to minimize the time required to achieve consensus in accordance with the network size. The results of our investigation showed that maintaining full coverage is the best strategy when the number of participating nodes is small. However, reducing the coverage to limit the number of neighbors is the best strategy for fast consensus when the number of participating nodes is greater than a certain value. In further research, we will study the consensus algorithm when considering other MAC protocols in various wireless network topologies.

6. Acknowledgments

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*Corresponding author: Hyun-Ho Choi, Ph.D.

Department of Electrical, Electronic and Control Engineering,

Institute for Information Technology Convergence,

Hankyong National University Anseong 456-749, Korea

E-mail: hhchoi@hknu.ac.kr