Hierarchical Interference Alignment for Downlink Heterogeneous Networks

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***Abstract*—This paper focuses on interference issues arising in the downlink of a heterogeneous network (HetNet), where small cells are deployed within a macrocell. Interference scenario in a HetNet varies based on the type of small cell access modes, which can be classified as either closed subscriber group (CSG) or open subscriber group (OSG) modes. For these two types of modes, we propose *hierarchical interference alignment* (HIA) schemes, which successively determine beamforming matrices for small cell and macro base stations (BSs) by considering a HetNet environment in which the macro BS and small cell BSs have different numbers of transmit antennas. Unlike prior work on interference alignment (IA) for homogeneous networks, the proposed HIA schemes compute the beamforming matrices in closed-form and reduce the feedforward overhead through a hierarchical approach. By providing a tight outer bound of the degrees-of-freedom (DoF), we also investigate the optimality of the proposed HIA schemes with respect to the number of antennas without any time expansion. Furthermore, we propose a new optimization process to maximize the sum-rate performance of each cell while satisfying the IA conditions. The simulation results show that the proposed HIA schemes provide an additional DoF compared to the conventional interference coordination schemes using a time domain-based resource partitioning. Under multi-cell interference environments, the proposed schemes offer an approximately 100% improvement in throughput gain compared to the conventional coordinated beamforming schemes when the interference from coordinated BSs is significantly stronger than the remaining interference from uncoordinated BSs.**

***Index Terms*—Interference alignment, interference management, heterogeneous network, sum rate maximization.**

I. Introduction

THE demand for greater mobile traffic in cellular networks is increasing exponentially; however, link efficiency is approaching its fundamental limit. To improve the spectral efficiency of cellular systems, heterogeneous network (HetNet) deployment, where low-power and small-coverage cells are distributed within the macrocell coverage, is considered to be a promising solution [1]-[8]. Small cells, such as pico- and femto-cells deployed at coverage holes or at capacity-demanding hotspots, can extend coverage and increase the spectral utilization. Moreover, they allow mobile stations to be closer to the base station (BS), which improves the received signal quality, potentially yielding an enhanced wireless capacity in cellular networks.

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Although the HetNet concept effectively improves the average spectral efficiency, its overlaid cell deployment with frequency reuse of one increases the probability of outages because users at the cell edges among the macrocell and small cells experience severe co-channel interference [3]-[5]. In addition, as the cell density increases because of an increased number of users, classical resource management techniques based on frequency/space reuse and power control are unable to cope with the additional interference. Therefore, interference management is critical for a successful deployment of small cells and a guaranteed quality-of-service (QoS) in cell edge areas.

To address the interference problem in HetNet, various interference coordination techniques including *coordinated scheduling* and *silencing* have been proposed [6]-[8]. These techniques perform adaptive resource partitioning in the time or frequency domain to cancel out strong interference and balance the load among co-existing cells. Interference management schemes relying on multiple antennas, such as coordinated beamforming and joint processing, have been considered by both academia and the industry [2], [9]. As an advanced beamforming technique, the idea of interference alignment (IA) has been proposed to manage interference by aligning multiple interference signals within a reduced dimensional subspace at each receiver. While most of the work on IA has focused on K point-to-point interfering links such as an X channel [10] and interference channel [11]-[15], it has also been shown in [16]-[19] that IA can be used to improve the user throughput at a cell-edge in cellular networks. In [20], the integration of IA with other system issues, such as opportunistic scheduling, has been proposed to mitigate inter-cell interference. However, there are few works addressing IA techniques for use in a HetNet. The IA technique was first applied to a HetNet environment in [21]; however, the optimized performance of this technique in generalized antenna settings and its system level performance in multi-cell environments have yet to be investigated.

The interference scenario in HetNet varies with the access mode of the small cell, which can be categorized into either closed subscriber group (CSG) or open subscriber group (OSG) mode [22]. In CSG mode, only authorized subscribers can attach to a small cell, and nonsubscribers are not always connected to the nearest BS. This creates strong interference components between different tiers and significantly degrades the performance of cell boundary users. In OSG mode, on the other hand, small cells are accessible to all users. Nevertheless, small cell attachment is not sufficient to attain solid cell splitting gains owing to the disparity between the transmit power of a macrocell and small cell. To solve this problem, an OSG cell uses a range expansion (RE) technique [4]. The RE technique increases the coverage of low-power small cells by adding a positive bias to their received signal strengths (RSS) during cell association. Accordingly, some macro users receiving interference from nearby small cell BSs turn into small cell users, which eventually mitigates inter-tier interference between small cell BS and macro users. Therefore, interference between a small cell BS and macro users becomes insignificant in OSG mode, while all interferences between two tiers are significant in CSG mode.

Herein, we consider a two-tier multiple-input multiple-output (MIMO) network in a downlink, consisting of a single macrocell BS with multiple transmit antennas and two MIMO picocells employed by either either CSG or OSG mode.[1] We propose *hierarchical interference alignment* (HIA) schemes by applying the concept of IA to a HetNet environment to mitigate both the inter-tier interference between a macrocell and picocells, and the inter-user interference between macro users. Two HIA schemes are developed according to the principal interference scenarios based on the two types of access modes, and an optimization process is provided to maximize the sum-rate performance of each cell while satisfying the IA conditions. We compare the proposed HIA schemes with conventional interference coordination schemes in terms of the degrees-of-freedom (DoF) and ergodic sum rate for an isolated cell layout. We also evaluate the system level performances of the proposed schemes for a realistic multi-cell layout.

The rest of this paper is organized as follows. We present our system model in Section II. In Section III, we describe our proposed HIA schemes in detail. In Section IV, we describe the optimization process used to maximize their performance. The performance results from each simulation scenario, are provided in Section V. In Section VI, we further introduce a generalized HIA and describe its feasibility condition in terms of the number of users and number of cells. Finally, Section VII provides some concluding remarks regarding the proposed schemes.

II. System Model

Fig. 1 shows the system model for a HetNet with MIMO antennas. There are two pico BSs (i.e., BS 1 and BS 3) and a macro BS (i.e., BS 2). The pico BS serves one user per cell and the macro BS serves two users simultaneously. Each user receives d independent streams along linearly independent beamforming vectors from its corresponding transmitter. As a typical antenna configuration, we assume that each pico BS and the macro BS are equipped with M and $2M$ transmit antennas, respectively, and all users have M receive antennas. We also suppose that perfect channel state information (CSI) is available at the transmitter and receiver.

The notations used in this paper are defined below:

[1]In general, operator-deployed cells, such as picocells, use OSG mode while user-deployed cells, such as femtocells, use CSG mode. However, to focus on the performance differences resulting from the access mode used, we regard picocells as small cells overlying macrocells and consider a situation in which the picocells can select one between two access modes.

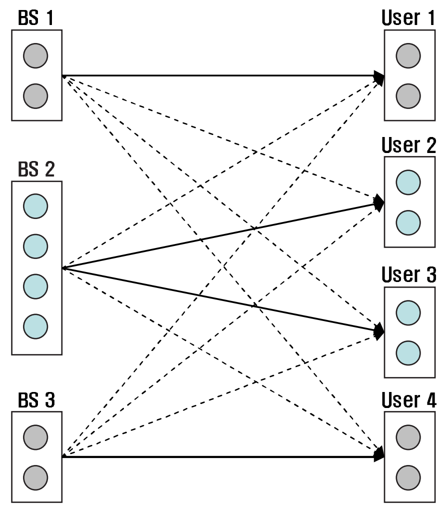


Fig. 1. System model for the MIMO heterogeneous network.

- $\mathbf{s}_k$: transmit symbol vector with a size of $d \times 1$ intended for user k denoted as $\mathbf{s}_k = [s_k^1 \ s_k^2 \cdots s_k^d]^T$.
- p_k: transmit power allocated to the k-th user's symbol vector with an average power constraint per BS, i.e., $p_1 = p_4 = P_{\text{pico}}$, $p_2 + p_3 = P_{\text{macro}}$, where P_{pico} and P_{macro} are the total transmit power at each pico BS and macro BS, respectively.
- $\mathbf{H}_{i,j}$: channel matrix from the j-th BS to the i-th user whose entry is independent and identically distributed according to $\mathcal{CN}(0,1)$, where $\mathbf{H}_{i,1} \in \mathbb{C}^{M\times M}$, $\mathbf{H}_{i,3} \in \mathbb{C}^{M\times M}$ and $\mathbf{H}_{i,2} \in \mathbb{C}^{M\times 2M}$ for all $i \in \{1,2,3,4\}$.
- $\mathbf{n}_k$: additive white Gaussian noise vector with a size of $M\times 1$ with variance σ^2 per entry observed at the receiver.
- $\mathbf{V}_k$: transmit beamforming matrix for the k-th user denoted as $\mathbf{V}_k = [\mathbf{v}_k^1 \ \mathbf{v}_k^2 \cdots \mathbf{v}_k^d]$ where $\mathbf{V}_1, \mathbf{V}_4 \in \mathbb{C}^{M\times d}$ and $\mathbf{V}_2, \mathbf{V}_3 \in \mathbb{C}^{2M\times d}$.
- $\mathbf{W}_k$: receive beamforming matrix with a size of $M \times d$ for the k-th user denoted as $\mathbf{W}_k = [\mathbf{w}_k^1 \ \mathbf{w}_k^2 \cdots \mathbf{w}_k^d]$.
- $(\cdot)^\dagger$: conjugate transpose operator.
- $(\cdot)^T$: transpose operator.
- $\{\cdot\} \setminus \{\cdot\}$: set difference operator.

The received signal at the k-th receiver is expressed as

$$\mathbf{y}_k = \sum_{m=1}^{4} \sqrt{p_m}\mathbf{H}_{k,\mathfrak{f}(m)}\mathbf{V}_m\mathbf{s}_m + \mathbf{n}_k \tag{1}$$

where $\mathfrak{f}(k)$ indicates the index of the serving BS of user k, such that $\mathfrak{f}(k) = 1,2,2,$ and 3 when $k = 1,2,3,$ and 4, respectively. Each user decodes the desired signals arriving from its corresponding BS by multiplying the receive beamforming matrix; hence, the signal at the user k after receiver combining is given by

$$\tilde{\mathbf{y}}_k = \mathbf{W}_k^\dagger \sum_{m=1}^{4} \sqrt{p_m}\mathbf{H}_{k,\mathfrak{f}(m)}\mathbf{V}_m\mathbf{s}_m + \tilde{\mathbf{n}}_k \tag{2}$$

where $\tilde{\mathbf{n}}_k = \mathbf{W}_k^\dagger\mathbf{n}_k$ is the effective noise vector with covariance $\sigma^2\mathbf{W}_k\mathbf{W}_k^\dagger$. For the given set of linear beamforming matrices, $\mathbf{V}_k$ and $\mathbf{W}_k$, where $k \in \{1,2,3,4\}$, the achievable

$$R_k = \sum_{i=1}^{d} \log \left(1 + \frac{p_k^i \left| \mathbf{w}_k^{i\dagger} \mathbf{H}_{k,\mathfrak{f}(k)} \mathbf{v}_k^i \right|^2}{\underbrace{\left| \mathbf{w}_k^{i\dagger} \sum_{j=1, j\neq i}^{d} \sqrt{p_k^j} \mathbf{H}_{k,\mathfrak{f}(k)} \mathbf{v}_k^j \right|^2}_{\text{intra-user interference}} + \underbrace{\left| \mathbf{w}_k^{i\dagger} \sum_{m=1, m\neq k}^{4} \sum_{j=1}^{d} \sqrt{p_m^j} \mathbf{H}_{k,\mathfrak{f}(m)} \mathbf{v}_m^j \right|^2}_{\text{inter-user interference}} + \sigma^2} \right) \quad (3)$$

rate at the user k is calculated as (3)[2], wherein p_k^i denotes the transmit power of the i-th transmit symbol for k-th user, s_k^i.

The DoF is a key metric to assess the system performance for a multiple antenna configuration in a high signal-to-noise ratio (SNR) regime. The DoF is defined as a pre-log factor of the sum rate, and is expressed as

$$d_\Sigma \triangleq \lim_{\mathsf{SNR}\to\infty} \frac{R_\Sigma(\mathsf{SNR})}{\log(\mathsf{SNR})} = \sum_{k=1}^{4} d_k \quad (4)$$

where $R_\Sigma(\mathsf{SNR}) = \sum_{k=1}^{4} R_k(\mathsf{SNR})$ denotes the sum rate at the $\mathsf{SNR} = P/\sigma^2$, and d_k is the individual DoF achieved by user k.

III. Hierarchical Interference Alignment

The motivation behind HIA is to exploit the heterogeneity between macrocells and picocells. That is, the number of transmit antennas, the transmission power, and the scale of users served differ considerably among different cells [1]-[3]. The key idea of HIA is to design transmit beamforming matrices sequentially in ascending order of the number of transmit antennas. More specifically, pico BSs, with a smaller number of transmit antennas, construct their beamforming matrices first so that interference vectors are aligned with a small dimensional space. Thereafter, the macro BS, with a larger number of transmit antennas, develops its beamforming matrices to align the interference vectors with the signal space spanned by the interference vectors caused by the predetermined beamforming matrices of the pico BSs. We develop two HIA schemes according to the two different interference scenarios of HetNet: CSG and OSG modes.

A. HIA for CSG mode

Fig. 2 illustrates the procedure used for designing of a beamforming matrix for CSG mode, where macro users receive considerable interference from nearby pico BSs. Here, the number of independent streams of each user, d, is equal to $\frac{M}{2}$. Two steps are required to determine the transmit and receive beamforming matrices (i.e., $\mathbf{V}_k$ and $\mathbf{W}_k$ where $k \in \{1, 2, 3, 4\}$).

[2]A non-linear beamforming strategy and a description of its achievable rate are given in Section IV.

1) Step 1: Design of beamforming matrices for Pico BSs: We first consider macro users 2 and 3. At users 2 and 3, the interference caused by the pico BSs should be aligned within an $M/2$ dimensional space to obtain $M/2$ interference-free dimensions from the M dimensional receive signal vector. Therefore, the following conditions are obtained:

$$\text{span}(\mathbf{H}_{2,1}\mathbf{V}_1) = \text{span}(\mathbf{H}_{2,3}\mathbf{V}_4), \quad (5)$$
$$\text{span}(\mathbf{H}_{3,1}\mathbf{V}_1) = \text{span}(\mathbf{H}_{3,3}\mathbf{V}_4) \quad (6)$$

where $\text{span}(\cdot)$ denotes the subspace spanned by the column vectors of a matrix. Owing to the fact that both $\mathbf{H}_{2,1}$ and $\mathbf{H}_{3,1}$ are invertible with a probability of one, (5) and (6) can be equivalently expressed as

$$\text{span}(\mathbf{V}_1) = \text{span}\left(\mathbf{H}_{2,1}^{-1}\mathbf{H}_{2,3}\mathbf{V}_4\right) = \text{span}\left(\mathbf{H}_{3,1}^{-1}\mathbf{H}_{3,3}\mathbf{V}_4\right). \quad (7)$$

To satisfy (7), by solving the generalized eigen-problem, we can find the transmit beamforming matrices $\mathbf{V}_1$ and $\mathbf{V}_4$ for the picocells as follows [23]:

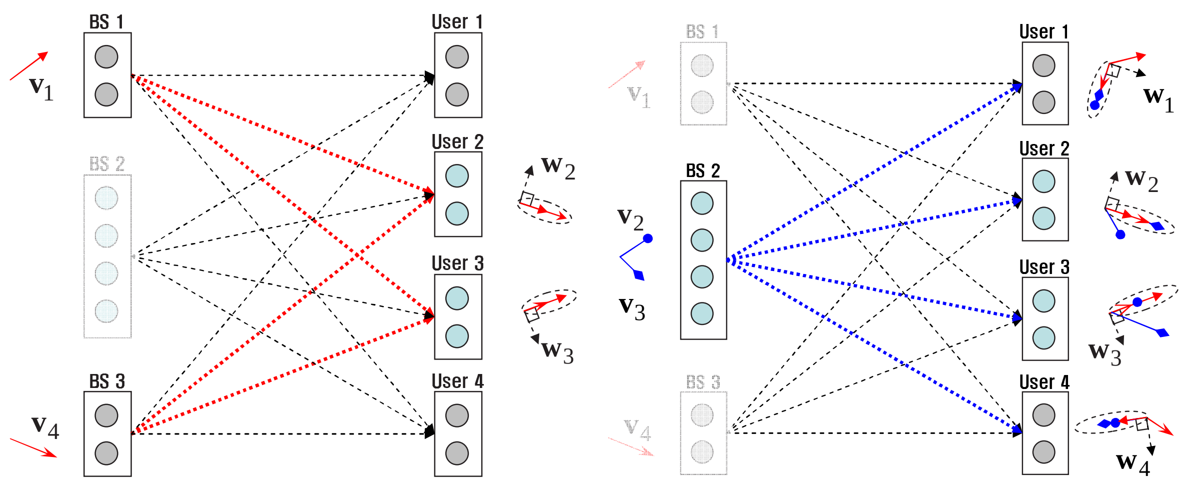
$$\mathbf{v}_4^i = \text{eig}\left(\mathbf{H}_{3,3}^{-1}\mathbf{H}_{3,1}\mathbf{H}_{2,1}^{-1}\mathbf{H}_{2,3}\right), \quad (8)$$
$$\mathbf{v}_1^i = \mathbf{H}_{3,1}^{-1}\mathbf{H}_{3,3}\mathbf{v}_4^i / \|\mathbf{H}_{3,1}^{-1}\mathbf{H}_{3,3}\mathbf{v}_4^i\| \quad (9)$$

where $i \in \{1, 2, \cdots \frac{M}{2}\}$, and $\text{eig}(\cdot)$ and $(\cdot)^{-1}$ denote a unit-norm eigenvector and the inverse of a matrix, respectively.

2) Step 2: Design of beamforming matrices for Macro BS: Before determining the transmit beamforming matrices of the macro BS, we design the receive beamforming matrices for all users in order to cancel out the interference signals from the pico BSs. Therefore,

$$\mathbf{W}_1 = \mathcal{N}((\mathbf{H}_{1,3}\mathbf{V}_4)^\dagger), \quad (10)$$
$$\mathbf{W}_4 = \mathcal{N}((\mathbf{H}_{4,1}\mathbf{V}_1)^\dagger), \quad (11)$$
$$\mathbf{W}_2 = \mathcal{N}((\mathbf{H}_{2,1}\mathbf{V}_1)^\dagger) = \mathcal{N}((\mathbf{H}_{2,3}\mathbf{V}_4)^\dagger), \quad (12)$$
$$\mathbf{W}_3 = \mathcal{N}((\mathbf{H}_{3,1}\mathbf{V}_1)^\dagger) = \mathcal{N}((\mathbf{H}_{3,3}\mathbf{V}_4)^\dagger) \quad (13)$$

where $\mathcal{N}(\cdot)$ denotes an orthonormal basis for the null space of a matrix. We can now find the transmit beamforming matrices for the macro BS such that all interference signals at the pico and macro users caused by the macro BS can be removed by applying transmit and receive beamforming matrices. In other words, the macro BS guarantees the transmitted signals of its corresponding users, $\mathbf{H}_{j2}\mathbf{V}_i, i \in \{2, 3\}$ to lie in the $M/2$ dimensional null space of $\mathbf{W}_j^\dagger$ for the non-intended j-th user, where $j \in \{1, 2, 3, 4\} \setminus \{i\}$. The IA conditions for the macro



(a) Design of beamforming matrices for pico BSs

(b) Design of beamforming matrices for macro BS

Fig. 2. HIA for CSG mode in HetNet ($M = 2$).

BS then can then be straightforwardly expressed as

$$\mathbf{V}_2 = \mathcal{N}\left(\left[\left(\mathbf{W}_1^\dagger \mathbf{H}_{1,2}\right)^\dagger \left(\mathbf{W}_4^\dagger \mathbf{H}_{4,2}\right)^\dagger \left(\mathbf{W}_3^\dagger \mathbf{H}_{3,2}\right)^\dagger\right]^\dagger\right), \quad (14)$$

$$\mathbf{V}_3 = \mathcal{N}\left(\left[\left(\mathbf{W}_1^\dagger \mathbf{H}_{1,2}\right)^\dagger \left(\mathbf{W}_4^\dagger \mathbf{H}_{4,2}\right)^\dagger \left(\mathbf{W}_2^\dagger \mathbf{H}_{2,2}\right)^\dagger\right]^\dagger\right). \quad (15)$$

Thus far, we demonstrated that every BS and user can eliminate all $\frac{3M}{2}$ interference signals at an unintended user using the linear beamforming matrices determined above. We now need to show whether each destination node can successfully decode $\frac{M}{2}$ desired data streams, i.e., a decodability check. Note that the direct channel matrices, $\mathbf{H}_{k,\mathfrak{f}(k)}$, do not appear in the interference alignment equations related to determination of $\mathbf{V}_k$. Therefore, the desired signal vectors at each user, $\mathbf{H}_{k,\mathfrak{f}(k)}\mathbf{V}_k$, are linearly independent of the interference signal vectors with a probability of one. This enables each user to decode $\frac{M}{2}$ desired data symbols using a zero forcing decoder $\mathbf{W}_k$. Therefore, the proposed HIA scheme determines all beamforming vectors that achieve $\frac{M}{2}$ DoF per user and $2M$ DoF in total.

Remark 1 (Optimality of HIA in CSG mode): To obtain an outer bound of the HetNet in CSG mode, we assume full cooperation in the exchange of both CSI and data among two pico BSs, pico users, and macro users. This is equivalent to a two-user $2M \times 2M$ MIMO interference channel. It is well known that the DoF for this channel is $2M$ [14], which coincides with the achievable DoF of the proposed HIA scheme. Hence, the proposed HIA for CSG mode is optimal in achieving the DoF without any time expansion.

B. HIA for OSG mode

Fig. 3 shows the procedure used for designing the beamforming matrix for OSG mode, where macro users receive negligible interference from the pico BSs owing to a range expansion. Although the transmit and receive beamforming vectors derived in Section III-A can be a solution for OSG mode, it is best to design beamforming vectors dedicated to OSG mode because of a smaller number of interference channel links. New transmit and receive matrices are first derived to maximize the desired signal power while concurrently satisfying the IA conditions. We assume that only one stream is transmitted to each mobile user, that is, $d = 1$. Thereafter, we prove that the proposed beamforming strategy achieves the full DoF solution for OSG mode with $M = 2$, but is suboptimal for other antenna configurations, when $M > 2$. The transmit and receive beamforming vectors are determined through the following two steps:

1) Step 1: Design of beamforming vectors for Pico BSs: To maximize the desired signal term, we assume that maximum ratio combining (MRC) is used at each user as follows [25]:

$$\mathbf{w}_k = \frac{\mathbf{H}_{k,\mathfrak{f}(k)}\mathbf{v}_k}{\|\mathbf{H}_{k,\mathfrak{f}(k)}\mathbf{v}_k\|}, \quad k = 1, 2, 3, 4. \quad (16)$$

From a zero other-cell interference constraint, we also have the following conditions:

$$\mathbf{w}_1^\dagger \mathbf{H}_{1,3}\mathbf{v}_4 = \mathbf{w}_4^\dagger \mathbf{H}_{4,1}\mathbf{v}_1 = 0. \quad (17)$$

Using (16), these conditions can be rewritten as

$$\mathbf{v}_1^\dagger \mathbf{H}_{1,1}^\dagger \mathbf{H}_{1,3}\mathbf{v}_4 = \mathbf{v}_4^\dagger \mathbf{H}_{4,3}^\dagger \mathbf{H}_{4,1}\mathbf{v}_1 = 0 \quad (18)$$

which implies that both $\mathbf{H}_{1,1}^\dagger \mathbf{H}_{1,3}\mathbf{v}_4$ and $\mathbf{H}_{4,1}^\dagger \mathbf{H}_{4,3}\mathbf{v}_4$ are in the null space of vector $\mathbf{v}_1$. Therefore,

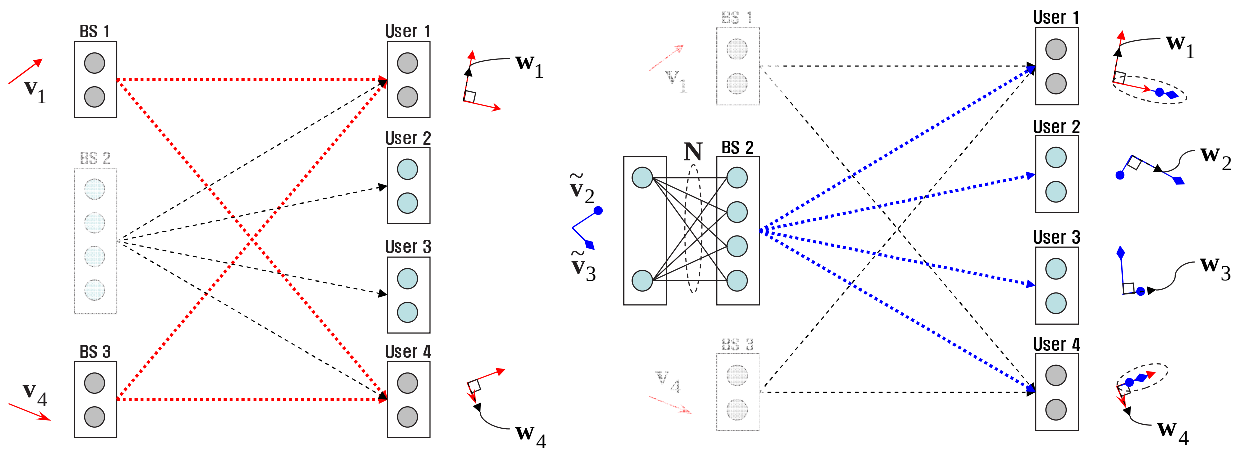
$$\mathcal{N}(\mathbf{v}_1) = \lambda_1 \mathbf{H}_{1,1}^\dagger \mathbf{H}_{1,3}\mathbf{v}_4 = \lambda_2 \mathbf{H}_{4,1}^\dagger \mathbf{H}_{4,3}\mathbf{v}_4 \quad (19)$$

$$\Leftrightarrow \mathbf{H}_{1,1}^\dagger \mathbf{H}_{1,3}\mathbf{v}_4 = \frac{\lambda_2}{\lambda_1}\mathbf{H}_{4,1}^\dagger \mathbf{H}_{4,3}\mathbf{v}_4. \quad (20)$$

This is known as a generalized eigen-problem, where λ_2/λ_1 is the generalized eigenvalue. Thus, we can find the transmit beamforming vectors of pico BSs, $\mathbf{v}_1$ and $\mathbf{v}_4$, that satisfy the above conditions as follows:

$$\mathbf{v}_4 = \text{eig}\left(\left(\mathbf{H}_{4,1}^\dagger \mathbf{H}_{4,3}\right)^{-1}\left(\mathbf{H}_{1,1}^\dagger \mathbf{H}_{1,3}\right)\right), \quad (21)$$

$$\mathbf{v}_1 = \mathcal{N}\left(\mathbf{v}_4^\dagger \mathbf{H}_{4,3}^\dagger \mathbf{H}_{4,1}\right). \quad (22)$$



(a) Design of beamforming matrices for pico BSs
(b) Design of beamforming matrices for macro BS

Fig. 3. HIA for OSG mode in HetNet ($M = 2$).

Given the transmit beamforming vectors $\mathbf{v}_1$ and $\mathbf{v}_4$, we can compute the receive beamforming vectors, $\mathbf{w}_1$ and $\mathbf{w}_4$, using (17).

2) Step 2: Design of beamforming vectors for Macro BS: The macro BS should design the transmit beamforming vectors $\mathbf{v}_2$ and $\mathbf{v}_3$ not to cause interference to the pico users who have already applied their receive beamforming vectors as determined through Step 1. Therefore, $\mathbf{v}_2$ and $\mathbf{v}_3$ should lie in the null space of the effective channels from the macro BS at the pico users as follows:

$$\mathbf{v}_k \prec \mathcal{N}\left(\underbrace{\left[(\mathbf{w}_1^\dagger \mathbf{H}_{1,2})^\dagger \;\; (\mathbf{w}_4^\dagger \mathbf{H}_{4,2})^\dagger\right]^\dagger}_{\mathbf{H}_{\text{eff}}}\right), \quad k \in \{2, 3\}, \tag{23}$$

where $\mathbf{H}_{\text{eff}}$ is a $2 \times 2M$ matrix and has a $2(M-1)$-dimensional null space where $M \geq 2$, and $\mathbf{A} \prec \mathbf{B}$ means that the set of column vectors of matrix $\mathbf{A}$ is a subset of the set of column vectors of matrix $\mathbf{B}$. Let us choose two arbitrary orthonormal $2M \times 1$ vectors, $\mathbf{n}_1$ and $\mathbf{n}_2$, in the null space. Then, $\mathbf{v}_k$ can be expressed as

$$\mathbf{v}_k = [\mathbf{n}_1 \;\; \mathbf{n}_2] \begin{bmatrix} \tilde{v}_k^1 \\ \tilde{v}_k^2 \end{bmatrix} \tag{24}$$

$$= \mathbf{N}\tilde{\mathbf{v}}_k \tag{25}$$

where $\mathbf{N}$ is a $2M \times 2$ matrix, and $\tilde{v}_k^1$ and $\tilde{v}_k^2$ are complex numbers. Thus, it is necessary to design a 2×1 vector $\tilde{\mathbf{v}}_k$ instead of designing a $2M \times 1$ vector $\mathbf{v}_k$. Equivalently, we can suppose that the macro BS loses $2(M-1)$ antennas, as shown in Fig. 3(b). In addition, to mitigate inter-user interference to macro users, we obtain the following conditions:

$$\mathbf{w}_2^\dagger \mathbf{H}_{2,2} \mathbf{v}_3 = \mathbf{w}_3^\dagger \mathbf{H}_{3,2} \mathbf{v}_2 = 0. \tag{26}$$

Using (16) and (25), this zero inter-user interference constraint is re-expressed as

$$\mathbf{v}_2^\dagger \mathbf{H}_{2,2}^\dagger \mathbf{H}_{2,2} \mathbf{v}_3 = \mathbf{v}_3^\dagger \mathbf{H}_{3,2}^\dagger \mathbf{H}_{3,2} \mathbf{v}_2 = 0 \tag{27}$$

$$\Leftrightarrow \tilde{\mathbf{v}}_2^\dagger \underbrace{\mathbf{N}^\dagger \mathbf{H}_{2,2}^\dagger \mathbf{H}_{2,2} \mathbf{N}}_{\mathbf{R}_{2,2}} \tilde{\mathbf{v}}_3 = \tilde{\mathbf{v}}_3^\dagger \underbrace{\mathbf{N}^\dagger \mathbf{H}_{3,2}^\dagger \mathbf{H}_{3,2} \mathbf{N}}_{\mathbf{R}_{3,2}} \tilde{\mathbf{v}}_2 = 0 \tag{28}$$

where $\mathbf{R}_{2,2}$ and $\mathbf{R}_{3,2}$ are 2×2 vectors. In the same manner as in (18)-(22), we can compute the transmit beamforming vectors for the macro BS, $\mathbf{v}_2$ and $\mathbf{v}_3$, by solving the generalized eigen-problem as follows:

$$\mathbf{v}_3 = \mathbf{N} \cdot \text{eig}\left(\left(\mathbf{R}_{3,2}^\dagger\right)^{-1} \cdot \mathbf{R}_{2,2}\right), \tag{29}$$

$$\mathbf{v}_2 = \mathbf{N} \cdot \mathcal{N}\left(\tilde{\mathbf{v}}_3^\dagger \mathbf{R}_{3,2}\right). \tag{30}$$

Note that we can compute the receive beamforming vectors $\mathbf{w}_2$ and $\mathbf{w}_3$ from (26) after obtaining the transmit beamforming vectors $\mathbf{v}_2$ and $\mathbf{v}_3$.

Remark 2 (Optimality and suboptimality of HIA in OSG mode): If we assume full cooperation among two pico BSs, pico users, and macro users, the HetNet in OSG mode can be modeled as a $2M \times 2M$ two-user MIMO Z channel. Recently, the DoF of this channel was recently determined to be $2M$ [26]. Since allowing transmitters and receivers to cooperate does not hurt the capacity, the DoF of the HetNet in OSG mode is no greater than $2M$. The proposed HIA for OSG mode specifically achieves the DoF of four, regardless of the value of M, because each user receives only one data stream from its corresponding BS. This means that the proposed HIA is the optimal DoF achieving solution for OSG mode when $M = 2$ and $d = 1$.

Remark 3 (Feedforward mechanism of HIA): The receive beamforming vectors are assumed to be MRC in order to maximize the serving signal strength with no inter-cell and inter-user interferences in OSG mode. It is possible to design the transmit and receive beamforming vectors in a different manner, such that both inter-cell interferences from adjacent BSs and inter-user interferences from serving BSs are aligned in the orthogonal direction of the serving signal at each user, which is illustrated in Fig. 3. Note that the receive beamforming vectors of MRC given by (16) are determined as a function of the effective channel from the serving BS after applying the corresponding transmit beamforming matrix. This enables each user itself to calculate its receive beamforming matrix without a *feedforward* mechanism from the BS (i.e., the delivery of the receive beamforming matrix predetermined at the serving BS through the downlink control channels). That is to say, each user estimates the effective channels from its serving BS, $\mathbf{H}_{k,\mathfrak{f}(k)}\mathbf{v}_k$, based on the demodulation reference signals defined in 3GPP LTE release 9 [1] and adopts the direction of the estimated effective channel as its receive beamforming matrix. Note that compared to most prior works on coordinated beamforming, which requires a feedforward mechanism [10]-[19], this operation significantly decreases the overhead of the control information through the use of physical downlink control channels (PDCCH).

IV. HIA-based Beamforming Matrix Optimization

In the previous section, we discussed how the proposed HIA schemes design the transmit and receive beamforming matrices to achieve the optimal DoF in the downlink MIMO channel of HetNet. Thus, it is not necessarily optimal in terms of the achievable rate region because zero-forcing criterions limit the spatial diversity order of the desired receivers. However, the fundamental question on the downlink channel is that, if the total amount of transmit power per BS is fixed, can we know the optimal capacity region for the sum rate? In addition, how can we design the transmit and receive beamforming matrices to achieve the optimal capacity region? To answer these questions, in this section, we describe a new suboptimal method for a beamforming matrix design that maximizes the achievable sum rate performance as well as the DoF metric. Our original problem can be formulated as

$$\max_{\mathbf{V}_i,\mathbf{W}_i,i\in\{1,2,3,4\}} \sum_{k=1}^{4} R_k \tag{31}$$

$$\text{s.t.} \quad \sum_{k=2}^{3} p_k = P_{\text{macro}}, \tag{32}$$

$$p_1 = p_4 = P_{\text{pico}}. \tag{33}$$

Since the derivation of the optimal beamforming matrices maximizing the achievable sum rate is a non-convex problem, the optimal solution may not be tractable in efficient manners. In spite of being suboptimal, to overcome this difficulty we first decouple the joint design problem into three sub-problems related to each cell and then determine the transmit and receive beamforming matrices to maximize the achievable per-cell sum rate as follows:

$$\max_{\mathbf{V}_i,\mathbf{W}_i,i\in\{1,2,3,4\}} \sum_{k=1}^{4} R_k \tag{34}$$

$$\text{s.t.} \quad \sum_{k=2}^{3} p_k = P_{\text{macro}}, \tag{35}$$

$$p_1 = p_4 = P_{\text{pico}}, \tag{36}$$

$$\mathbf{W}_i\mathbf{H}_{k,\mathfrak{f}(k)}\mathbf{V}_k = \mathbf{0},\ \forall \mathfrak{f}(i) \neq \mathfrak{f}(k). \tag{37}$$

To facilitate this decoupling, we apply the IA methods as the initial step for inter-cell interference mitigation. From the zero inter-cell interference constraint, the transmit and receive beamforming matrices for the pico BSs and pico users are designed as,

$$\mathbf{V}_1^{\text{opt}} = \mathbf{V}_1 \cdot \widetilde{\mathbf{V}}_1 \quad \text{and} \quad \mathbf{V}_4^{\text{opt}} = \mathbf{V}_4 \cdot \widetilde{\mathbf{V}}_4, \tag{38}$$

$$\mathbf{W}_1^{\text{opt}} = \mathbf{W}_1 \cdot \widetilde{\mathbf{W}}_1 \quad \text{and} \quad \mathbf{W}_4^{\text{opt}} = \mathbf{W}_4 \cdot \widetilde{\mathbf{W}}_4 \tag{39}$$

where $\mathbf{V}_k$ and $\mathbf{W}_k$ are the subspaces that ensure the IA conditions for the inter-cell interference derived in the previous section, and $\widetilde{\mathbf{V}}_k = \left[\widetilde{\mathbf{v}}_k^1, \widetilde{\mathbf{v}}_k^2, \cdots \widetilde{\mathbf{v}}_k^{\frac{M}{2}}\right]$ and $\widetilde{\mathbf{W}}_k = \left[\widetilde{\mathbf{w}}_k^1, \widetilde{\mathbf{w}}_k^2, \cdots \widetilde{\mathbf{w}}_k^{\frac{M}{2}}\right]$ are the matrices that optimally combine the column spaces of $\mathbf{V}_k$ and $\mathbf{W}_k$, respectively, in order to maximize the achievable per-cell sum rate.

To guarantee zero out-of-cell interference, the transmit and receive beamforming matrices for the macrocell are designed in a manner similar to that for the picocells:

$$\mathbf{V}_2^{\text{opt}} = \mathbf{V}_{\text{macro}} \cdot \widetilde{\mathbf{V}}_2 \quad \text{and} \quad \mathbf{V}_3^{\text{opt}} = \mathbf{V}_{\text{macro}} \cdot \widetilde{\mathbf{V}}_3, \tag{40}$$

$$\mathbf{W}_2^{\text{opt}} = \mathbf{W}_2 \cdot \widetilde{\mathbf{W}}_2 \quad \text{and} \quad \mathbf{W}_3^{\text{opt}} = \mathbf{W}_3 \cdot \widetilde{\mathbf{W}}_3 \tag{41}$$

where $\mathbf{V}_{\text{macro}}$ should lie within the null space of the effective channels from the macro BS to the pico users in order to ensure no inter-cell interference from the macro BS. It then follows:

$$\mathbf{V}_{\text{macro}} = \mathcal{N}\left(\left[\left(\mathbf{W}_1^{\dagger}\mathbf{H}_{1,2}\right)^{\dagger} \ \left(\mathbf{W}_4^{\dagger}\mathbf{H}_{4,2}\right)^{\dagger}\right]^{\dagger}\right). \tag{42}$$

We now optimize the beamforming weights for pico users with respect to the individual rate maximization under the zero inter-cell interference constraint. Applying (38)-(41) to (3), the achievable sum rate for user 1 and 4 in picocells can be written as

$$R_k = \sum_{i=1}^{\frac{M}{2}} \log\left(1 + \frac{p_k^i\left|\widetilde{\mathbf{w}}_k^{i,\dagger}\widetilde{\mathbf{H}}_{k,\mathfrak{f}(k)}\widetilde{\mathbf{v}}_k^i\right|^2}{\underbrace{\left|\widetilde{\mathbf{w}}_k^{i\dagger}\sum_{j=1,j\neq i}^{\frac{M}{2}}\sqrt{p_k^i}\widetilde{\mathbf{H}}_{k,\mathfrak{f}(k)}\widetilde{\mathbf{v}}_k^j\right|^2}_{\text{intra}-\text{stream interference}} + \sigma^2}\right), \quad i \in \{1,4\} \tag{43}$$

where $\widetilde{\mathbf{H}}_{i,\mathfrak{f}(i)}$ denotes the effective channel from BS $\mathfrak{f}(i)$ to the user i, which can be written as

$$\widetilde{\mathbf{H}}_{i,\mathfrak{f}(i)} = \mathbf{W}_i^{\dagger}\mathbf{H}_{i,\mathfrak{f}(i)}\mathbf{V}_{\text{macro}},\ \ i \in \{1,4\}. \tag{44}$$

In (43), there is no inter-cell interference term from the other picocells and the macro BS because the transmit and receive beamforming matrices in (38)-(41) were designed to ensure the zero inter-cell interference for all pico and macro users. Hence, the original problem becomes equivalent to determining the optimal weight matrix $\widetilde{\mathbf{V}}_i$ and $\widetilde{\mathbf{W}}_i$ to achieve the maximum achievable sum rate performance in an effective single-user MIMO channel, the channel matrix of which is $\widetilde{\mathbf{H}}_{i,\mathfrak{f}(i)}, i \in \{1,4\}$. The optimal transmit and receive beamforming technique, given a perfect CSI in a single-user MIMO, is the singular value decomposition (SVD) precoder [24], which can be calculated as

$$\widetilde{\mathbf{H}}_{i,\mathfrak{f}(i)} = \mathbf{U}_{i,\mathfrak{f}(i)}\mathbf{\Sigma}_{i,\mathfrak{f}(i)}\mathbf{F}_{i,\mathfrak{f}(i)}^{\dagger}. \tag{45}$$

The optimal weighting matrix for the picocells is then set as

$$\widetilde{\mathbf{V}}_i = \mathbf{F}_{i,\mathfrak{f}(i)} \quad \text{and} \quad \widetilde{\mathbf{W}}_i = \mathbf{U}_{i,\mathfrak{f}(i)}^{\dagger}, \quad i \in \{1,4\} \tag{46}$$

and the power allocation between the spatial layers is determined by waterfilling over the diagonal values of the matrix, $\boldsymbol{\Sigma}_{i,\mathfrak{f}(i)}$, corresponding to the signal power delivered for each spatial layer.

We next explain the procedure used for designing the beamforming matrices for macro users to maximize their sum rate. While IA is applied only for inter-cell interference, we apply dirty paper coding (DPC) [27], [28], a well-known technique for achieving the capacity region of a multiuser MIMO broadcast channel, to deal with the remaining inter-user interference for macro users. Therefore, the sum rate maximization problem for both users in the macro BS is expressed as

$$\max_{\boldsymbol{\Sigma}_i}\sum_{k=2}^{3} R_k = \max_{\widetilde{\mathbf{V}}_i}\sum_{k=2}^{3}\log\frac{\left|\mathbf{I}+\widetilde{\mathbf{H}}_{k,\mathfrak{f}(k)}\left(\sum_{i=2}^{k}\boldsymbol{\Sigma}_i\right)\widetilde{\mathbf{H}}_{i,\mathfrak{f}(i)}^{\dagger}\right|}{\left|\mathbf{I}+\widetilde{\mathbf{H}}_{i,\mathfrak{f}(i)}\left(\sum_{i=2}^{k-1}\boldsymbol{\Sigma}_i\right)\widetilde{\mathbf{H}}_{k,\mathfrak{f}(k)}^{\dagger}\right|} \quad (47)$$

$$\text{s.t.} \quad \sum_{k=2}^{3}\operatorname{tr}\left(\boldsymbol{\Sigma}_k\right) \le P_{\text{macro}} \quad (48)$$

where $\widetilde{\mathbf{H}}_{i,\mathfrak{f}(i)}$ is written as

$$\widetilde{\mathbf{H}}_{i,\mathfrak{f}(i)} = \mathbf{W}_i^{\dagger}\mathbf{H}_{i,\mathfrak{f}(i)}\mathbf{V}_{\text{macro}}, \quad i \in \{2,3\} \quad (49)$$

and $\boldsymbol{\Sigma}_i$ is the virtual transmit covariance matrix for the i-th user in the macro BS, $\boldsymbol{\Sigma}_i \succeq 0$, i.e., $\boldsymbol{\Sigma}_i$ is a semidefinite matrix, defined as

$$\boldsymbol{\Sigma}_i = \widetilde{\mathbf{V}}_i\mathbb{E}\left\{\mathbf{s}_i\mathbf{s}_i^{\dagger}\right\}\widetilde{\mathbf{V}}_i^{\dagger}, \quad i \in \{2,3\}. \quad (50)$$

By applying the principle of multiple-access channel-broadcast channel (MAC-BC) duality [29], we can solve the MIMO sum rate optimization problem. This problem is transformed into a dual MAC problem, which can be solved using existing iterative water-filling based algorithms [30], [31] or a sub-gradient algorithm [32]. This process is straightforward, and we therefore omit its details in this paper. Consequently, the optimal beamforming strategy under zero inter-cell interference constraints for all users is completely determined.

V. Result and Discussion

We compare the proposed HIA schemes with two interference management schemes: time division multiple access (TDMA) and TDMA with interference aware coordinated beamforming (IA-CBF) [25]. As a baseline scheme, we also consider a non-coordinated beamforming without other-cell interference control (non-CBF w/o IC). In the TDMA scheme, different time slots are allocated in each cell ensuring that there is no inter-cell interference among the macro and picocells. Each pico BS transmits M data streams to the intended user based on eigen beamforming, and the macro BS transmits M data streams to each user using a multi-user MIMO technique (i.e., block diagonalization) [33]. In the TDMA with IA-CBF, the data transmission is performed using only two time slots. In the first time slot, pico BSs transmit $\frac{M}{2}$ data streams to the corresponding user simultaneously, whereby the interference signals coming from the other pico BS are canceled out, while the macro BS remains silent. In the second time slot, the macro BS operates in multi-user MIMO mode in the same manner as that of the TDMA case, and thus each macro user can receive M data streams from the macro BS at the same time.

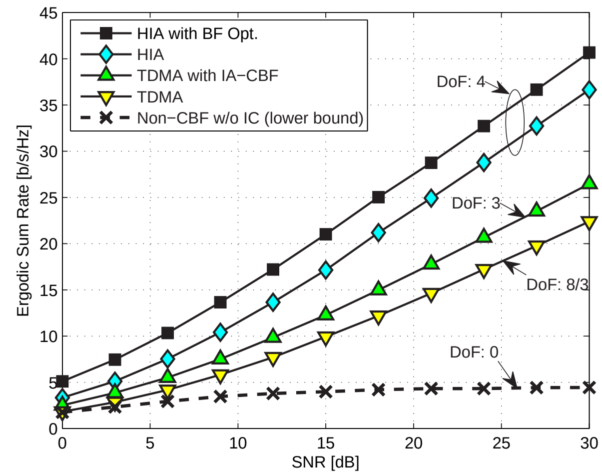


Fig. 4. Achievable ergodic sum rate vs. SNR for $M = 2$ and CSG mode.

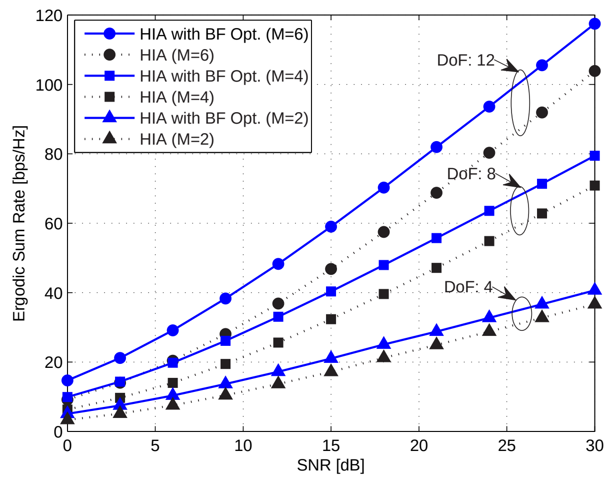


Fig. 5. Achievable ergodic sum rate according to the number of antennas.

A. Performances in isolated three-cell layout

Fig. 4 shows the achievable ergodic sum rate versus the SNR in an isolated cell layout for $M = 2$ and CSG mode (i.e., the RE technique is not used). The proposed HIA schemes outperform the conventional schemes in terms of the sum rate and the DoF. We observe that the proposed HIA schemes achieve the optimal DoF of 4, while the DoFs of the TDMA and the TDMA with IA-CBF are $\frac{8}{3}$ and 3, respectively. Note that the HIA with beamforming optimization shows better performance because it further maximizes the per-cell sum rate performance while retaining the optimal DoF achieved by HIA. We can claim that the proposed system achieves considerably better performance than the other existing solutions, especially in a high SNR regime.

Fig. 5 shows the achievable ergodic sum rate of the two proposed HIA schemes for CSG mode according to the

TABLE I
SIMULATION SETUP

Parameter	Assumption
Macrocell layout	Hexagonal 19 cells, 3 sectors per cell
Inter-BS distance	500 m
Number of MSs per sector	30 (uniform distribution)
Carrier frequency / Bandwidth	2 GHz / 10 MHz
Macro BS transmission power	46 dBm
Pico BS transmission power	30 dBm
Path loss from macro BS to user	128.1+37.6$log_{10}R$ [dB], R in km
Path loss from pico BS to user	140.7+36.7$log_{10}R$ [dB], R in km
Macrocell antenna pattern	3D pattern of Table A.2.1.1-2 in [1]
Picocell antenna pattern	0 dB (omni-directional)
Channel model	ITU-R M.1255 Ped. A & 3GPP SCM
Shadowing standard deviation	8 dB
Penetration Loss	20 dB
Noise figure	9 dB
Traffic model	full buffer

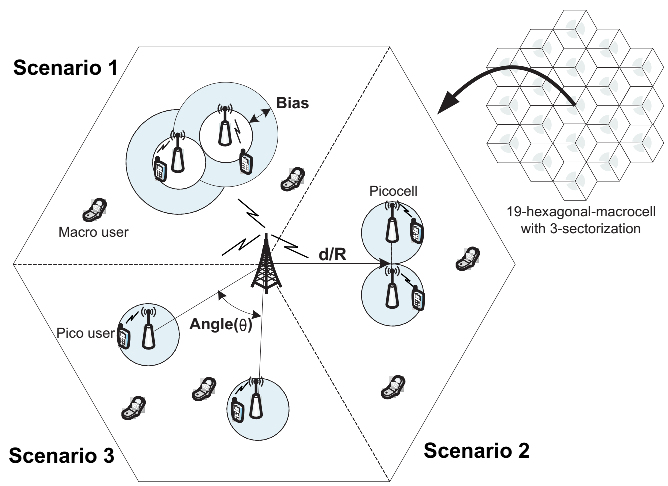


Fig. 6. Simulation scenarios in multi-cell layout.

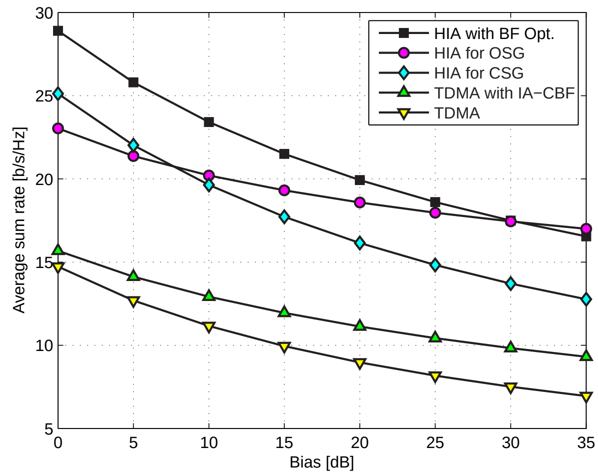


Fig. 7. Scenario 1: average sum rate vs. bias value.

number of antennas. The sum rate increases linearly with a slope of 4, 8, and 12 for $M = 2, 4$, and 6, respectively. We verify that the sum rate performance coincides with the DoF of $2M$, as proven in Sections III. It is also observed that performance gain from the beamforming optimization increases as the number of antennas increases.

B. Performances in multi-cell layout

We further perform a system level simulation in a multi-cell layout. Nineteen hexagonal macrocells with three-sectorization are used, and two picocells are installed in one sector of the center macrocell. As shown in Fig. 6, we examine the performance dynamics of three scenarios, and find the best condition for applying the proposed HIA scheme to a realistic multi-cell environment. The coverage of the picocells is varied with the bias value, and the positions of the pico BSs are changed according to the distance between the macro BS and pico BS ($\frac{d}{R}$), or the angle between two pico BSs (θ). The mobile users are deployed in a uniform-random manner only in the center cell area, and users participating in the HIA operation are randomly chosen in each cell. Our simulation conforms to the evaluation methodology of 3GPP [1]. Table I summarizes the simulation parameters and assumptions.

Fig. 7 shows the average sum rate according to the bias value when $\frac{d}{R}$=0.45 and θ=20°. Using the RE technique, we add a bias to the RSS of each picocell to extend its coverage. Therefore, as the bias value increase, the picocell coverage enlarge, and more users are attached to the picocell. However, this decreases the average SNR of the attached pico users because the average distance between the pico BS and the pico users increases, thereby decreasing the data rate of the picocells. At low bias, the interference from the pico BSs is not negligible for macro users, and thus the HIA for OSG mode, which does not treat the interference from the pico BSs, shows worse performance than the HIA for CSG mode, which aligns the interferences from the pico BSs. As the bias increases, however, the interference from the pico BSs to the macro users becomes negligible. This validates the assumption of HIA for OSG mode and thus the HIA for OSG mode, which performs MRC to maximize the desired signal strength under the assumption of no interference from a pico BS, outperforms the HIA for CSG mode, as well as the HIA with BF optimization at a very high bias.

Fig. 8 shows the average sum rate according to the relative distance between the macro BS and pico BSs as a function of $\frac{d}{R}$, where R is the radius of the macrocell when the bias is 20 dB and θ=20°. When the pico BSs move into the inner area of the cell (i.e., $\frac{d}{R}$ decreases), the SINR of the pico users is reduced owing to severe interference from the adjacent sectors. Moreover, when the pico BSs are located in the outer area of the cell, the SINR decreases because of severe interference from neighboring macrocells. When the pico BSs are located in the cell's center area, interference from their coordinated macro BS is very strong; however, interference from the other macro BSs is not strong because of their three-dimensional antenna pattern. Since the proposed HIA schemes cancel out the interference from the coordinated macro BS, the pico BSs located in the middle area, with a $\frac{d}{R}$ of approximately 0.45, receive the least interference from the other uncoordinated BSs; they exhibit the best performance with an approximately 100% improvement over the conventional schemes.

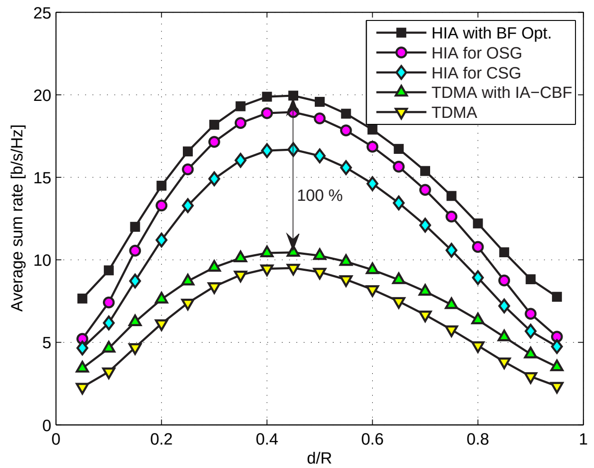


Fig. 8. Scenario 2: average sum rate vs. relative distance between macro BS and pico BSs ($\frac{d}{R}$).

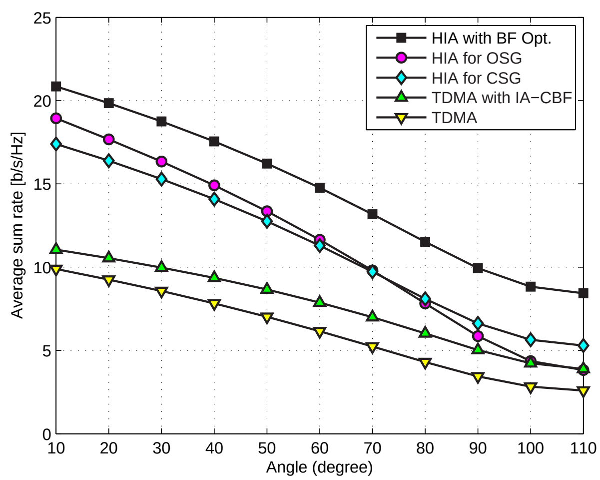


Fig. 9. Scenario 3: average sum rate vs. angle between two pico BSs.

Fig. 9 shows the average sum rate according to the angle between two pico BSs when the bias is 20 dB and $\frac{d}{R}$ is 0.45. As the angle between the two coordinated pico BSs increases, the pico BSs approach the cell boundary and receive more interference from the neighboring macro BSs. This SINR degradation eventually decreases the data rate of the pico users. In addition, as the angle between two pico BSs increases, the area interfered by two picocells becomes wider, and therefore macro users receive greater interference from two pico BSs on average. In this type of situation, a bias of 20 dB is not sufficient to mitigate the interference from a pico BS to a macro user, and therefore at a large angle, the performance of HIA for OSG mode, which does not treat the interference from a pico BS to macro users, becomes worse than that of HIA for CSG mode.

VI. Extension to Generalized HIA

To extend the proposed HIA, this section is concerned with general parameters: K_p pairs of pico BS and pico user (i.e., BSs 1, 2, $\cdots$, K_p and users 1, 2, $\cdots$, K_p) and K_m macro users (i.e., users K_p+1, K_p+2, $\cdots$, K_p+K_m) in a single macro BS (i.e., BS K_p+1). Let N_p^t and N_m^t denote the number of transmit antennas at each pico BS and macro BS, respectively. We assume that all users are equipped with N^r receive antennas. According the whether the IA is feasible or not, the system can be either proper or improper [15]. To establish new feasibility conditions for the generalized HIA, the number of equations and variables are investigated based on Bezout's theorem, and the relationship among the number of antennas at each pico BS, macro BS, and macro users is given as the basic requirement for a proper system.

Proposition 1: For a generalized HetNet, the total $(K_p+K_m)\,d$ DoF (i.e., d DoF per user) is achieved by using a generalized HIA technique as long as

$$N_p^t+\kappa N^r \geq \kappa(K_p+1)d \quad \text{and} \quad N_m^t \geq \kappa K_p d \tag{51}$$

where $\kappa=\frac{K_m}{K_p}+1$.

Proof: We divide the generalized HIA into two steps similar to the HIA described in Section III. First, to guarantee zero inter-cell interference from the pico BS, we jointly construct all transmit and receive beamforming matrices for the pico cells, and receive beamforming matrices for the macro users. The conditions are given by

$$\mathbf{W}_i^\dagger \mathbf{H}_{i,\mathfrak{f}(k)}\mathbf{V}_k=\mathbf{0},\ \forall k\neq i\in\{1,\cdots,K_p\}, \tag{52}$$

$$\mathbf{W}_j^\dagger \mathbf{H}_{j,\mathfrak{f}(k)}\mathbf{V}_k=\mathbf{0},\ \forall k\in\{1,\cdots,K_p\}\ \text{and}\ \forall j\in\{K_p+1,\cdots,K_p+K_m\}, \tag{53}$$

$$\mathrm{rank}(\mathbf{W}_i^\dagger \mathbf{H}_{i,\mathfrak{f}(i)}\mathbf{V}_i)=d,\ \forall i\in\{1,\cdots,K_p\}. \tag{54}$$

In the general user setting, it is not straightforward to obtain a closed-form solution by simply extending the HIA because the HIA is originated from the generalized eigen-problem for the special case, $K_p = K_m = 2$ as presented in Section III. The transmit beamforming matrices for a pico BS should be constructed such that not only all interferences caused by all pico BSs are confined to a small interference subspace at each macro user by (53), but also all inter pico-cell interference signals are aligned at each pico user by (52), unlike the case for $K_p = K_m = 2$. Note that the inter pico-cell interference alignment condition in (52) is not necessary for the case of $K_p = 2$ since one inter pico-cell interference only exists from the other pico BS. Our approach here is to consider the signal space interference alignment problem as the solvability of a multivariate polynomial system [15]. In fact, the generalized HIA method requires iterative computations [15] to find a solution in the first step while the HIA schemes do not require it.[3] The IA conditions of (52) and (53) contain $N_e=(K_p+K_m-1)K_p d$ equations with $N_v = K_p(N_p^t+N^r)+K_m N^r$ variables. However, similar to the IA conditions for the interference network considered in [15], many of these variables are superfluous owing to the conditions required for a linear independence of

[3]Note that it is difficult to derive closed-form solutions for even the interference alignment for MIMO homogeneous interference channels with more than three users. Computing transmit/receive beamforming matrices for a generalized HIA with a closed-form solution remains an open problem, and we thus leave it for future work.

the desired signals (54). Hence, the addition of the variables to the transmit and receive sides provides the total number of independent variables in the system as follows:

$$\overline{N}_v = K_p(N_p^t + N^r - 2d) + K_m(N^r - d). \quad (55)$$

If the channel coefficients are i.i.d. across all BSs and users, then the existence of an IA solution is guaranteed by Bezout's theorem almost surely, which requires $\overline{N}_v \geq N_e$. Therefore,

$$N_p^t + \kappa N^r \geq \kappa(K_p + 1)d. \quad (56)$$

In the second step, the transmit beamforming matrix is designed under the other transmit/receive beamforming matrices given in the first step, and thus all inter-cell interferences among pico users and all inter-user interferences among macro users are completely eliminated. This idea is based only on simple zero-forcing transmit beamforming, and therfore requires the second feasibility condition of (51). ∎

Remark 4 (Comparison with previous results): When $K_m = K_p = 2$ (i.e., $\kappa = 2$), (51) is reduced to

$$N_p^t + 2N^r \geq 6d \text{ and } N_m^t \geq 4d. \quad (57)$$

Note that our constructive HIA methods, as described in Section III, provide closed-form solutions achieving total $(K_p + K_m)\, d = 4 \times \frac{M}{2} = 2M$ DoF for $N_p^t = N^r = M$ and $N_m^t = 2M$, exactly satisfying the feasibility condition in (57).

VII. Conclusions

We proposed HIA schemes for HetNet with macro and pico BSs by considering two different picocell access modes: OSG and CSG. The proposed HIA schemes follow the strategy of a two-stage beamforming design, which is motivated by the fact that a pico BS generally has fewer antennas than a macro BS. The proposed HIA schemes compute the beamforming matrices in a closed-form and reduce the control signaling overhead in the system. Moreover, we developed an HIA-based beamforming matrix optimization method for per-cell sum rate maximization. The analysis and simulation results show that the proposed HIA schemes achieve the optimal DoF, and their sum rates are significantly improved through beamforming optimization. Moreover, the multi-cell simulation results show that an appropriate HIA mode should be chosen adaptively according to the bias value to maximize the HIA gain. In addition, the HIA schemes are suitably applicable when the coordinated picocells were adjacently located in the middle of the cell, where the interference from uncoordinated BSs was the least.

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