Interfering Channel Alignment and Degrees of Freedom for Downlink Multicell MIMO Networks

Wonjae Shin, *Member, IEEE* Jong-Bu Lim, *Member, IEEE*, Changyong Shin *Member, IEEE*
Hyun-Ho Choi, *Member, IEEE*, and KyungHun Jang, *Member, IEEE*

***Abstract*—To solve the multi-cell interference problem in downlink cellular networks, we study G-cell multiple input multiple output Gaussian interfering broadcast channels (MIMO-IFBC) with K users on the cell-boundary of each base station (BS). The main idea is to align interfering channels from each BS to its non-intended users onto a small dimensional subspace. This is called *interfering channel alignment (ICA)*. We analytically derive both the feasibility conditions for the ICA to achieve a total number of degrees of freedom (DoF) of GK and the maximum number of DoF, which is achieved by the ICA for a given antenna conf guration. The numerical results show that our analysis is valid and the achievable DoF of the proposed ICA outperforms that of conventional schemes.**

***Index Terms*—Interference alignment, degrees of freedom.**

I. Introduction

INTERFERENCE management has become a key challenge in the design of future wireless cellular networks to satisfy the demand for higher data rates. In order to increase the system capacity for multi-cell and multi-user downlink transmissions, interference management techniques, such as coordinated multipoint transmission and reception (CoMP), have recently been highlighted [1]. However, due to the nature of broadcast and superposition over the wireless medium, each user suffers from inter-user interference (IUI) and inter-cell interference (ICI), as each base station (BS) sends independent messages to different users in each cell.

To remove these interferences, a simple zero-forcing (ZF) scheme in [2] simultaneously mitigates both ICI and IUI in two-cell multiple-input multiple-output interfering broadcast channels (MIMO-IFBC). It is shown that when there are two BSs with M transmit antennas and K users with N receive antennas in each cell, the ZF scheme in the two-cell MIMO-IFBC achieves a number of degrees of freedom (DoF) of $\min\{2M, 2KN, \max(M,N)\}$. In [3], a precoding scheme called subspace interference alignment (IA) for two-cell MIMO-IFBC was introduced. The subspace IA is composed of two cascaded precoders. The f rst precoder puts ICI vectors into a f nite multi-dimensional subspace. The second precoder is constructed so that the IUI vectors lie on the subspace spanned by the ICI vectors, which increases the subspace dimension for the desired signal vectors particulary in the case of a symmetric antenna conf guration. Furthermore, there are some further results on the DoF obtained by IA in the two-cell case [5]-[7]. It is shown that in [5] the DoF per cell is $\frac{KM}{K+\min(M,K)}$ for two-cell MISO-IFBC by assuming time or frequency extensions. [6] has proved that K DoF per cell can be achieved in two-cell K-user MIMO-IFBC if $M = K + 1$ and $N = K$. By providing a tight DoF outer bound, [7] has investigated the optimality of the IA in the two-cell network. An IA solution for the three-cell case has been recently presented in [8]. However, the scheme does not appear to guarantee higher DoF gains than the existing resource partitioning method in the specif c cases because some of the precious dimensions are wasted at the BSs.

Since the previous studies dealt with two or three-cell scenarios, there is still no closed-form solution for general MIMO cellular networks. In this letter, we consider G-cell, K-user MIMO networks and propose a new linear beamforming scheme, termed *interfering channel alignment (ICA)*. The main idea is to align multiple interfering channels from each BS to its non-intended users onto a small dimensional subspace by employing the intersection subspace property of vector space. With the detailed operation of the ICA scheme, the feasibility conditions for the proposed ICA are characterized in terms of the number of cells and the number of users in each cell. Furthermore, we investigate how many cell-edge users per cell (K) can be served simultaneously to maximize the total DoF achievable with a f xed number of antennas.

II. System Model

We describe a system model for the multi-cell MIMO-IFBC. The system contains G cells. Each cell has one BS and K users, i.e., mobile stations (MSs). The k-th MS in the i-th cell is denoted as $[k, i]$ for $k \in \{1, 2, \cdots, K\}$ and $i \in \{1, 2, \cdots, G\}$. Each BS is equipped with M antennas, and each MS is equipped with N antennas.

We def ne some parameters and notations as follows:

- $\mathbf{H}_j^{[k,i]}$: $N \times M$ channel matrix from the BS j to the MS $[k, i]$, of which entry is independent and identically distributed (i.i.d.) with $\mathcal{CN}(0, 1)$.
- $s^{[k,i]}$: symbol transmitted to the k-th MS in the i-th cell with an average power constraint P.
- $\mathbf{v}^{[k,i]}$: $M \times 1$ transmit beamforming vector for carrying the symbol $s^{[k,i]}$ with the unit norm $\|\mathbf{v}^{[k,i]}\| = 1$.
- $\mathbf{w}^{[k,i]}$: $N \times 1$ receive beamforming vector at the MS $[k, i]$ with the unit norm $\|\mathbf{w}^{[k,i]}\| = 1$.
- $\mathbf{n}^{[k,i]}$: $N \times 1$ additive white Gaussian noise (AWGN) vector at the MS $[k, i]$ with variance σ^2 per entry.
- $(\cdot)^\dagger$: conjugate transpose operator.

W. Shin, J.-B. Lim, C. Shin, and K. Jang are with Signal and Systems Lab., Samsung Advanced Institute of Technology (SAIT), Samsung Electronics, Korea (e-mail: {wonjae.shin, jongbu.lim, c.y.shin, khjang}@samsung.com).

H.-H. Choi is with the Department of Electrical, Electronic, and Control Engineering, Hankyong National University, Korea (e-mail: hhchoi@knuu.ac.kr).

$$R^{[k,i]} = \log_2\left(1+\frac{\left|\mathbf{w}^{[k,i]\dagger}\mathbf{H}_i^{[k,i]}\mathbf{v}^{[k,i]}\right|^2 P}{\sum_{k'=1,k'\neq k}^{K}\left|\mathbf{w}^{[k,i]\dagger}\mathbf{H}_i^{[k,i]}\mathbf{v}^{[k',i]}\right|^2 P+\sum_{i'=1,i'\neq i}^{G}\sum_{k'=1}^{K}\left|\mathbf{w}^{[k,i]\dagger}\mathbf{H}_{i'}^{[k,i]}\mathbf{v}^{[k',i']}\right|^2 P+\sigma^2}\right) \quad (1)$$

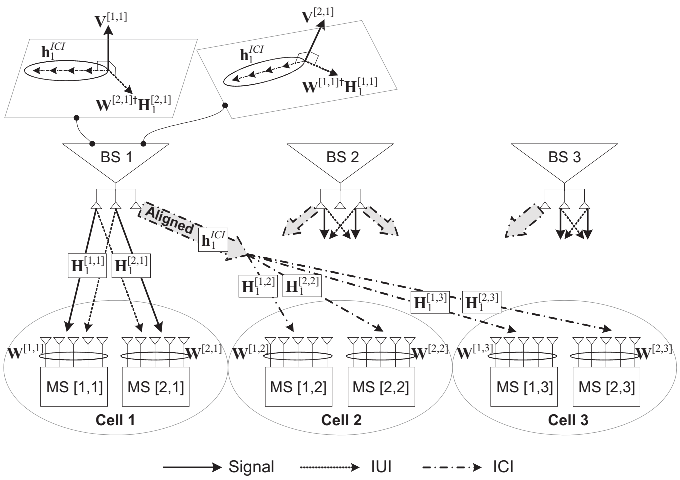

Fig. 1. Proposed ICA for $(G,K,M,N) = (3,2,3,5)$.

The received signal $\mathbf{y}^{[k,i]}$ at the MS $[k,i]$ is represented as

$$\begin{aligned}\mathbf{y}^{[k,i]} &= \underbrace{\mathbf{H}_i^{[k,i]}\mathbf{v}^{[k,i]}s^{[k,i]}}_{\text{desired signal}} + \sum_{k'=1,k'\neq k}^{K}\underbrace{\mathbf{H}_i^{[k,i]}\mathbf{v}^{[k',i]}s^{[k',i]}}_{\text{IUI}} \\ &+ \sum_{i'=1,i'\neq i}^{G}\sum_{k'=1}^{K}\underbrace{\mathbf{H}_{i'}^{[k,i]}\mathbf{v}^{[k',i']}s^{[k',i']}}_{\text{ICI}} + \mathbf{n}^{[k,i]}. \end{aligned} \quad (2)$$

After each MS decodes its desired signal by applying receive beamforming, the signal at the MS $[k,i]$ is expressed as

$$\begin{aligned}\tilde{y}^{[k,i]} &= \mathbf{w}^{[k,i]\dagger}\mathbf{H}_i^{[k,i]}\mathbf{v}^{[k,i]}s^{[k,i]} \\ &+ \mathbf{w}^{[k,i]\dagger}\sum_{k'=1,k'\neq k}^{K}\mathbf{H}_i^{[k,i]}\mathbf{v}^{[k',i]}s^{[k',i]} \\ &+ \mathbf{w}^{[k,i]\dagger}\sum_{i'=1,i'\neq i}^{G}\sum_{k'=1}^{K}\mathbf{H}_{i'}^{[k,i]}\mathbf{v}^{[k',i']}s^{[k',i']} + \tilde{n}^{[k,i]}\end{aligned} \quad (3)$$

where $\tilde{n}^{[k,i]} = \mathbf{w}^{[k,i]\dagger}\mathbf{n}^{[k,i]}$ is the effective noise. The achievable rate at the MS $[k,i]$ is obtained as (1).

The DoF (also known as the multiplexing gain) is defined as a pre-log factor of the sum rate. The individual DoF achieved by MS $[k,i]$ and the total DoF are expressed as, respectively,

$$d^{[k,i]} \triangleq \lim_{\gamma\to\infty}\frac{R^{[k,i]}(\gamma)}{\log(\gamma)} \quad \text{and} \quad d_\Sigma = \sum_k\sum_i d^{[k,i]} \quad (4)$$

where the SNR γ is given by $\frac{P}{\sigma^2}$.

III. Interfering Channel Alignment

The proposed ICA scheme operates with two steps. First, each MS cooperatively constructs the receive beamforming vectors in order to align the effective ICI channel within a small dimensional space. Through this ICI channel alignment, each BS can regard $K(G-1)$ different ICI channels as Δ-dimensional ICI channel vectors with $\Delta \leq K(G-1)$. Second, each BS removes ICI and IUI simultaneously by making the transmit beamforming vector orthogonal to the subspace spanned by the effective ICI channel and IUI channel vectors. Here, we assume that the perfect knowledge of channel state information (CSI) is available at both the BS and the MS.

In order to facilitate understanding of the ICA scheme, we first discuss one example with $(G,K,M,N) = (3,2,3,5)$ as illustrated in Fig. 1, and then extend it to the general case.

A. Example Case of $(G,K,M,N) = (3,2,3,5)$

1) Step 1. Design of receive beamforming vectors through ICI channel alignment: The receive beamforming vectors of MSs $[1,i]$ and $[2,i]$, i.e., $\mathbf{w}^{[1,i]}$ and $\mathbf{w}^{[2,i]}$, are determined in order that the effective ICI channels from the BS i to all non-intended MSs in the other cell are aligned, where $i \in \{1,2,3\}$. The ICI channel alignment conditions in each BS are given by

$$\begin{aligned}\text{span}\left(\mathbf{h}_1^{ICI}\right) &= \text{span}\left(\mathbf{H}_1^{[1,2]\dagger}\mathbf{w}^{[1,2]}\right) = \text{span}\left(\mathbf{H}_1^{[2,2]\dagger}\mathbf{w}^{[2,2]}\right) \\ &= \text{span}\left(\mathbf{H}_1^{[1,3]\dagger}\mathbf{w}^{[1,3]}\right) = \text{span}\left(\mathbf{H}_1^{[2,3]\dagger}\mathbf{w}^{[2,3]}\right),\end{aligned} \quad (5)$$

$$\begin{aligned}\text{span}\left(\mathbf{h}_2^{ICI}\right) &= \text{span}\left(\mathbf{H}_2^{[1,1]\dagger}\mathbf{w}^{[1,1]}\right) = \text{span}\left(\mathbf{H}_2^{[2,1]\dagger}\mathbf{w}^{[2,1]}\right) \\ &= \text{span}\left(\mathbf{H}_2^{[1,3]\dagger}\mathbf{w}^{[1,3]}\right) = \text{span}\left(\mathbf{H}_2^{[2,3]\dagger}\mathbf{w}^{[2,3]}\right),\end{aligned} \quad (6)$$

$$\begin{aligned}\text{span}\left(\mathbf{h}_3^{ICI}\right) &= \text{span}\left(\mathbf{H}_3^{[1,1]\dagger}\mathbf{w}^{[1,1]}\right) = \text{span}\left(\mathbf{H}_3^{[2,1]\dagger}\mathbf{w}^{[2,1]}\right) \\ &= \text{span}\left(\mathbf{H}_3^{[1,2]\dagger}\mathbf{w}^{[1,2]}\right) = \text{span}\left(\mathbf{H}_3^{[2,2]\dagger}\mathbf{w}^{[2,2]}\right),\end{aligned} \quad (7)$$

where $\text{span}(\cdot)$ means the space spanned by the column vectors of a matrix, and $\mathbf{h}_i^{ICI}$means the basis vector of aligned effective interference channels after applying the receive beamforming vectors to all the interfered MSs that suffer from ICI by the interfering BS i. Note that in this operation, the received interferences are not aligned from the viewpoint of a MS, but the interfering channels are aligned from the viewpoint of a BS. As more restrictive conditions, (5)-(7) can be rewritten by

$$\begin{aligned}\mathbf{h}_1^{ICI} &= \mathbf{H}_1^{[1,2]\dagger}\mathbf{w}^{[1,2]} = \mathbf{H}_1^{[2,2]\dagger}\mathbf{w}^{[2,2]} \\ &= \mathbf{H}_1^{[1,3]\dagger}\mathbf{w}^{[1,3]} = \mathbf{H}_1^{[2,3]\dagger}\mathbf{w}^{[2,3]}\end{aligned} \quad (8)$$

$$\begin{aligned}\mathbf{h}_2^{ICI} &= \mathbf{H}_2^{[1,1]\dagger}\mathbf{w}^{[1,1]} = \mathbf{H}_2^{[2,1]\dagger}\mathbf{w}^{[2,1]} \\ &= \mathbf{H}_2^{[1,3]\dagger}\mathbf{w}^{[1,3]} = \mathbf{H}_2^{[2,3]\dagger}\mathbf{w}^{[2,3]}\end{aligned} \quad (9)$$

$$\begin{aligned}\mathbf{h}_3^{ICI} &= \mathbf{H}_3^{[1,1]\dagger}\mathbf{w}^{[1,1]} = \mathbf{H}_3^{[2,1]\dagger}\mathbf{w}^{[2,1]} \\ &= \mathbf{H}_3^{[1,2]\dagger}\mathbf{w}^{[1,2]} = \mathbf{H}_3^{[2,2]\dagger}\mathbf{w}^{[2,2]}\end{aligned} \quad (10)$$

To jointly construct the received beamforming vectors, we aggregate the ICI channel alignment conditions of all BSs given by (8)-(10) into a unified system equation, as follows:

$$\begin{bmatrix} \mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{H}_1^{[1,2]\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{H}_1^{[2,2]\dagger} & \mathbf{0} & \mathbf{0} \\ \mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{H}_1^{[1,3]\dagger} & \mathbf{0} \\ \mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{H}_1^{[2,3]\dagger} \\ \mathbf{0} & \mathbf{I}_M & \mathbf{0} & -\mathbf{H}_2^{[1,1]\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_M & \mathbf{0} & \mathbf{0} & -\mathbf{H}_2^{[2,1]\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{H}_2^{[1,3]\dagger} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{H}_2^{[2,3]\dagger} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_M & -\mathbf{H}_3^{[1,1]\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_M & \mathbf{0} & -\mathbf{H}_3^{[2,1]\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_M & \mathbf{0} & \mathbf{0} & -\mathbf{H}_3^{[1,2]\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{H}_3^{[2,2]\dagger} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1^{ICI} \\ \mathbf{h}_2^{ICI} \\ \mathbf{h}_3^{ICI} \\ \mathbf{w}^{[1,1]} \\ \mathbf{w}^{[2,1]} \\ \mathbf{w}^{[1,2]} \\ \mathbf{w}^{[2,2]} \\ \mathbf{w}^{[1,3]} \\ \mathbf{w}^{[2,3]} \end{bmatrix} = \mathbf{F}\mathbf{x} = \mathbf{0} \tag{11}$$

where $\mathbf{F}$ is a $12M \times (3M + 6N)$ matrix with orthogonal columns. From (11), we can obtain the receive beamforming vectors that satisfy (8)-(10) by finding the null space of $\mathbf{F}$. It should be noted that there exists a vector $\mathbf{x}$ in the null space of $\mathbf{F}$ only if the column size of $\mathbf{F}$ is greater than the row size of $\mathbf{F}$ by the DoF of each MS, $d^{[k,i]} = 1$. Therefore, $3M + 6N > 12M$ should be satisfied. In this case of M=3 and N=5, the size of $\mathbf{F}$ is 36 $\times$ 39. Thus, it has a three-dimensional null space and the receive beamforming vectors for ICI channel alignment is obtained explicitly with probability one.

2) Step 2. Design of transmit beamforming vectors through IUI and ICI cancellations: Since the effective ICI channels are aligned with each other, the BS i can regard four different ICI channel vectors as a single ICI channel vector that spans one dimensional subspace, as shown in Fig. 1. Therefore, BS i can send the symbols $s^{[1,i]}$ and $s^{[2,i]}$ to MS $[1,i]$ and MS $[2,i]$, respectively, using only three transmit antennas without any interference. To completely eliminate interference, the transmit beamforming vectors for two MSs in the BS i, $\mathbf{v}^{[1,i]}$ and $\mathbf{v}^{[2,i]}$, are designed as

$$\mathbf{v}^{[1,i]} \subset \mathrm{null}\left(\left[(\mathbf{w}^{[2,i]\dagger}\mathbf{H}_i^{[2,i]})^\dagger, \mathbf{h}_i^{ICI}\right]^\dagger\right), \tag{12}$$

$$\mathbf{v}^{[2,i]} \subset \mathrm{null}\left(\left[(\mathbf{w}^{[1,i]\dagger}\mathbf{H}_i^{[1,i]})^\dagger, \mathbf{h}_i^{ICI}\right]^\dagger\right), \tag{13}$$

where $\mathrm{null}(\cdot)$ denotes an orthonormal basis for the null space of a matrix.

B. General Case

On the basis of the result of the previous example, we extend the proposed ICA scheme to the general case with G cells and K MSs. The main result is summarized as follows:

Theorem 1: For the G-cell MIMO-IFBC where each BS supports K MSs, we can achieve one DoF per MS if

$$M \geq K + \Delta \quad \text{and} \quad N > \frac{\{(G-1)K - \Delta\}M}{K}, \tag{14}$$

where $\Delta \in \{0, 1, \cdots, \min[(G-1)K, M-1]\}$ is the dimension of the aligned effective ICI channels.

Proof: In order to align $K(G-1)$ effective ICIs within Δ-dimensional space, the receive beamforming vectors should be constructed as

$$\begin{aligned} &\mathrm{span}\left(\left[\mathbf{h}_{j,1}^{ICI}, \mathbf{h}_{j,2}^{ICI}, \cdots, \mathbf{h}_{j,\Delta}^{ICI}\right]\right) = \\ &\mathrm{span}\left(\left[\mathbf{H}_j^{ICI,1}, \cdots, \mathbf{H}_j^{ICI,j-1}, \mathbf{H}_j^{ICI,j+1}, \cdots, \mathbf{H}_j^{ICI,G}\right]\right) \end{aligned} \tag{15}$$

where $\mathbf{H}_j^{ICI,i} = \left[\mathbf{H}_j^{[1,i]\dagger}\mathbf{w}^{[1,i]}, \mathbf{H}_j^{[2,i]\dagger}\mathbf{w}^{[2,i]}, \cdots, \mathbf{H}_j^{[K,i]\dagger}\mathbf{w}^{[K,i]}\right]$ is a channel matrix composed of K ICI vectors from the j-th BS to all K MSs in the i-th cell after applying the receive beamforming vectors for $i \neq j$.

To minimize the required number of transmit and receive antennas (M and N), we need to restrict the condition (15) and can represent it as a matrix form in a similar way to (11). Thus, we can generate a new unified system matrix $\mathbf{F}$ with a size of $G(G-1)MK \times (GM\Delta + GKN)$ and obtain the receive beamforming vectors by finding the null space of $\mathbf{F}$. The existence condition of the null space of $\mathbf{F}$ is given by

$$N > \frac{\{(G-1)K - \Delta\}M}{K}. \tag{16}$$

Note that since $N > 0$ should be satisfied, Δ should be smaller than $(G-1)K$. Furthermore, if all the effective ICI channels to each MS in interfered cells after applying receive beamforming vectors are aligned within a different Δ-dimension space, the minimum number of transmit antennas needed to eliminate both ICI and IUI simultaneously is given by

$$M \geq K + \Delta. \tag{17}$$

where since $K \geq 1$, $\Delta \leq M - 1$. Finally, the minimum number of antennas required at the BS and the MS is given by, respectively,

$$M^* = K + \Delta \quad \text{and} \quad N^* = \left\lceil \frac{\{(G-1)K - \Delta\}M}{K} + \epsilon \right\rceil. \tag{18}$$

where ϵ is an arbitrarily small positive number, $\epsilon \ll 1$. ■

C. Achievable DoF

In cellular networks, generally the number of installed antennas (i.e., M and N) is fixed and not large due to the limited physical size of the devices. However, the numbers of cooperative cells and users (i.e., G and K, respectively) are variable according to scheduling and their ranges are much wider than those of M and N. In particular, there are many MSs interfered by neighboring cells owing to the practically overlaid cell structure, and the condition on the value of K is less tight than the one on the value of G. Therefore, we try to derive the maximum K value when each of K MSs in a cell has one-DoF at a given (G, M, N) condition. This eventually leads to the maximization of achievable DoF. In *Theorem 1*, the condition (14) is represented as

$$\frac{KN}{(G-1)K - \Delta} > M \geq K + \Delta. \tag{19}$$

From (19), we have two inequalities as follows:

$$K \leq M - \Delta, \tag{20}$$

$$K < \frac{N-(G-2)\Delta + \sqrt{\{N-(G-2)\Delta\}^2 + 4(G-1)\Delta^2}}{2(G-1)}. \tag{21}$$

By combining (20) and (21), K is bounded as

$$K \le \min\{M-\Delta, \left\lfloor \frac{N-(G-2)\Delta+\sqrt{\{N-(G-2)\Delta\}^2+4(G-1)\Delta^2}}{2(G-1)} - \epsilon \right\rfloor\}, \quad (22)$$

where $\Delta \in \{0, 1, \cdots, \min[(G-1)K, M-1]\}$.

Let us define $f_1(\Delta) = M - \Delta$ and $f_2(\Delta) = \frac{N-(G-2)\Delta+\sqrt{\{N-(G-2)\Delta\}^2+4(G-1)\Delta^2}}{2(G-1)}$. Then, our objective function is described as

$$\max_{\Delta} K = \max_{\Delta} \min\{f_1(\Delta), \lfloor f_2(\Delta) - \epsilon \rfloor\} \quad (23)$$

$$\text{s. t.} \quad G, M, N > 1, \quad (24)$$

$$\Delta \in \{0, 1, \cdots, \min[(G-1)K, M-1]\}. \quad (25)$$

Note that $f_1(\Delta)$ is a decreasing function and $f_2(\Delta)$ is a convex function for Δ because of $\frac{d^2 f_2}{d\Delta^2} = \frac{2N^2}{\{4N\Delta+(N-G\Delta)^2\}^{3/2}} > 0$. Thus, the maximum K can be found at either $\Delta^* = 0$ or $\lceil \alpha \rceil$ where α is derived from the equation $f_1(\alpha) = f_2(\alpha)$ and is given by $\alpha = \frac{(G-1)M^2-MN}{GM-N}$. First, at $\Delta^* = \lceil \alpha \rceil$, a local maximum K is given by

$$K_1 = f_1(\lceil \alpha \rceil) = M - \left\lceil \frac{(G-1)M^2-MN}{GM-N} \right\rceil = \left\lfloor \frac{M^2}{GM-N} - \epsilon \right\rfloor. \quad (26)$$

Then, at $\Delta^* = 0$, the other local maximum K is given by

$$K_2 = \min\left\{M, \left\lfloor \frac{N}{G-1} - \epsilon \right\rfloor\right\}. \quad (27)$$

Therefore, the maximum K value, K^*, is expressed as

$$K^* = \max\{K_1, K_2\}$$
$$= \max\left\{\left\lfloor \frac{M^2}{GM-N} - \epsilon \right\rfloor, \min\left\{M, \left\lfloor \frac{N}{G-1} - \epsilon \right\rfloor\right\}\right\} \quad (28)$$

We observe that the condition $K_1 \ge K_2$ is equivalent to $M \ge N$. Therefore, the solution of the optimization problem is summarized as follows.

If $M \ge N$,

$$\Delta^* = \left\lceil \frac{(G-1)M^2-MN}{GM-N} \right\rceil, \; K^* = \left\lfloor \frac{M^2}{GM-N} - \epsilon \right\rfloor \quad (29)$$

If $M < N$,

$$\Delta^* = 0, \; K^* = \min\left\{M, \left\lfloor \frac{N}{G-1} - \epsilon \right\rfloor\right\}. \quad (30)$$

Note that Δ^* denotes the optimal dimension parameter that we should control to maximize the DoF, and K^* means the maximum number of MSs in a cell which have the DoF of one (i.e., the optimal DoF per cell) by using Δ^*-dimension for ICI alignment. In practice, we can choose Δ^* and then find K^* MSs for cooperation at a given (G, M, N) condition in order to achieve the maximum DoF in MIMO-IFBC.

Remark 1: When $M < N$, the ICA corresponds to multi-user MIMO to deal with IUI from a BS perspective while each user eliminates the remaining ICIs by decoding all the messages in the network and discarding out-of-cell users' signals.

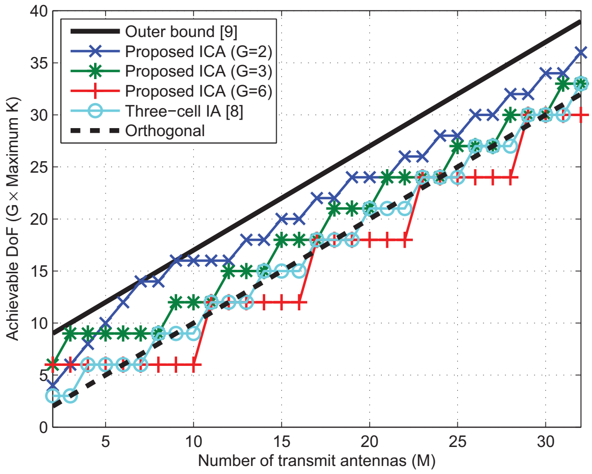

Fig. 2. Achievable DoF vs. number of transmit antennas when N=8.

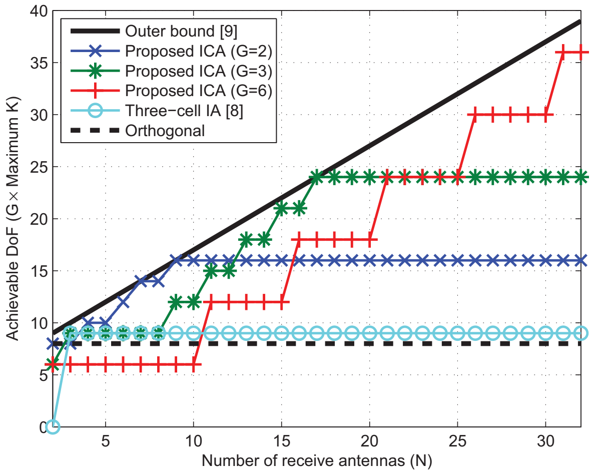

Fig. 3. Achievable DoF vs. number of receive antennas when M=8.

IV. Result and Discussion

We compare the proposed ICA scheme with the orthogonal scheme (e.g. resource partitioning among interfering BSs), three-cell IA [8], and outer bound from [9]. We choose G=2, 3, and 6 by considering a cell structure with a maximum of six sectors.

Fig. 2 shows the achievable DoF versus the number of transmit antennas when N is fixed to 8. As M increases, the achievable DoF of all schemes increase. In general, the performance of the proposed ICA scheme is reduced with an increase in the number of cooperating cells, G, because the number of ICIs increases. When $G \le 3$, the proposed ICA is overall better than the conventional schemes. This is attributed to the fact that the proposed ICA exploits the signal space more efficiently and searches for the optimal ICI alignment space within the wider range of $1 \sim (G-1)K$.

Fig. 3 demonstrates the achievable DoF versus the number of receive antennas when M is set to 8. While the conventional schemes exhibit constant performances, the DoF of the ICA

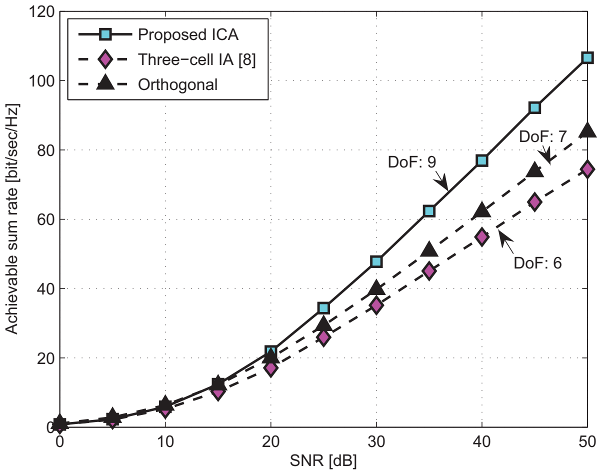

Fig. 4. Ergodic sum rate performance comparison for $(G, K, M, N) = (3, 3, 7, 5)$

scheme increases with N. In this case, the increasing rate is slightly better than tha in the case when N is f xed and M increases, as shown in Fig. 2. However, the performance of the ICA scheme is eventually limited to GM owing to (30) if N increases by a certain point.

In Fig. 4, we demonstrate the ergodic sum rate performance of the proposed scheme compared with those of existing schemes. As shown in the f gure, the proposed ICA scheme exhibits superior performance to others in case of $(G, K, M, N) = (3, 3, 7, 5)$. We also observe that the sum rate of the proposed ICA scheme increases linearly with the slope of 9 while those of the existing schemes increase linearly with the slope of 6 and 7. From this results, it conf rms that the sum rate performances exactly coincide with the DoF results which is analytically derived in Section III.

V. Conclusion

We proposed ICA scheme for downlink multi-cell MIMO networks, jointly designing transmit and receive beamforming vectors. The ICA scheme is inspired by the intersection subspace property of the vector space, thereby exploiting the signal space eff ciently. We derived the minimum antenna conf guration in order to achieve one DoF per user. From a practical perspective, we also characterized the maximum number of scheduled users through the ICA for any antenna conf guration. By the downlink and uplink duality, the degrees of freedom results in this paper are also applicable to the uplink system.

References

[1] *3GPP TR 36.819*, "*Coordinated multi-point operation for LTE physical layer aspects (Release 11)*," Dec. 2011.

[2] J. Kim, S.-H. Park, H. Sung, and I. Lee, "Sum rate analysis of two-cell MIMO broadcast channels: spatial multiplexing gain," *Proc. IEEE ICC*, Cape Town, South Africa, May 2010.

[3] C. Suh, M. Ho, and D. Tse, "Downlink interference alignment," *IEEE Trans. Communications*, vol. 59, no. 9, pp. 2616-2626, Sept. 2011.

[4] V. R. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the K-User interference channel," *IEEE Trans. Inf. Theory,* vol. 54, no. 8, pp. 3425-3441, Aug. 2008.

[5] J. Sun, Y. Liu, and G. Zhu, "On the degrees of freedom of the cellular network," *Proc. IEEE ICC*, Cape Town, South Africa, May 2010.

[6] W. Shin, N. Lee, J.-B. Lim, C. Shin, and K. Jang, "Interference alignment through user cooperation for two-cell MIMO interfering broadcast channels," *Proc. IEEE GLOBECOM*, Miami, U.S.A., Dec. 2010.

[7] T. Kim, D.J. Love, B. Clerckx, and D. Hwang, "Spatial degrees of freedom of the multicell MIMO multiple access channel," *Proc. IEEE GLOBECOM*, Houston, U.S.A., Dec. 2011.

[8] Y. Ma, J. Li, and R. Chen, "On the achievability of interference alignment for three-cell constant cellular interfering networks," *IEEE Communications Letters*, vol. 16, no. 9, pp. 1384-1387, Sept. 2012.

[9] B. Zhuang, R. A. Berry, and M. L Honig, "Interference alignment in mimo cellular networks," *Proc. IEEE ICASSP*, May 2011

[10] G. Golub and C. V. Loan, "Matrix computation," *The Johns Hopkins Press*, Baltimore, Maryland, second edition , 1989.